Statistics of Wave Fields in Complex Enclosures in the Frequency and Time Domains

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12 July, 2011

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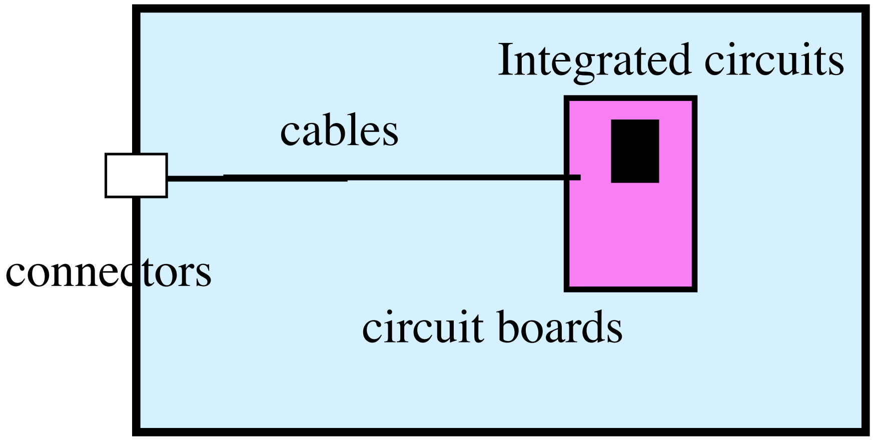
DEPARTMENT OF
ELECTRICAL & COMPUTER ENGINEERING
A. JAMES CLARK SCHOOL of ENGINEERING

Supported by AFOSR, ONR

Motivation: Electromagnetic Coupling to Electronic Circuits in Enclosures

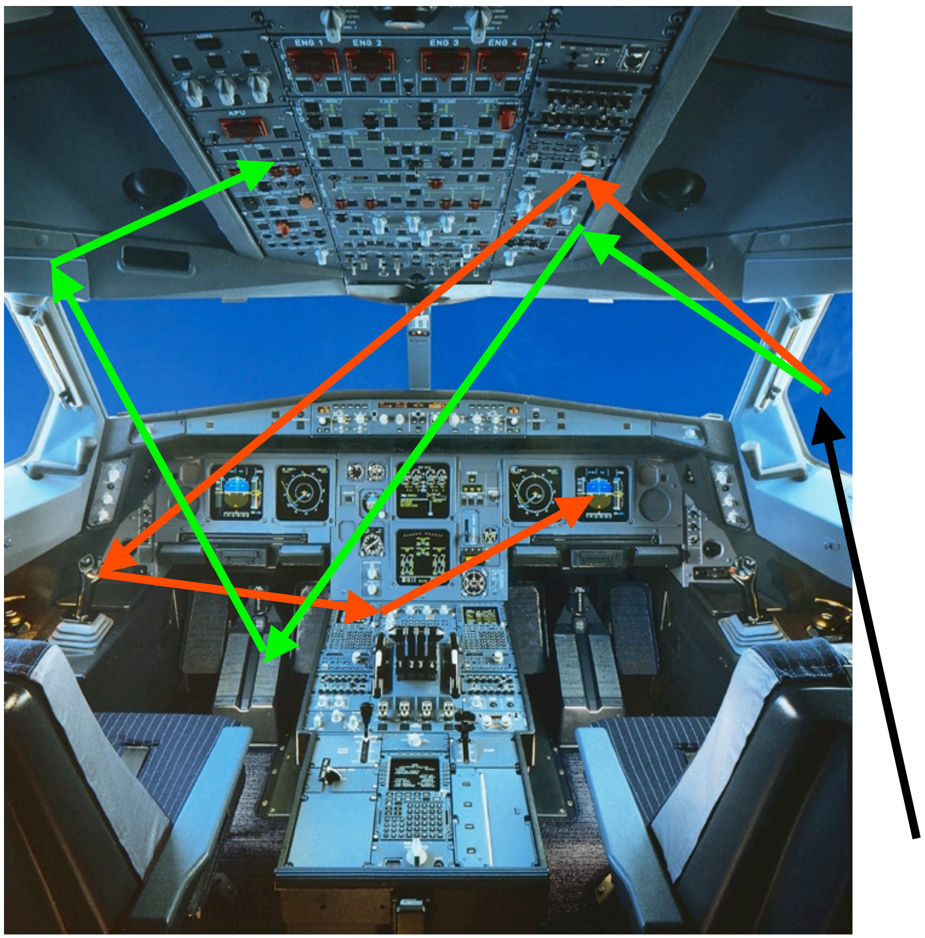
Schematic



- Coupling of external radiation to computer circuits is a complex processes:
  - apertures
  - resonant cavities
  - transmission lines
  - circuit elements
- Intermediate frequency range involves many interacting resonances
- System size >>Wavelength
- Chaotic Ray Trajectories
- "Wave Chaos"

- What can be said about coupling without solving in detail the complicated EM problem ?
- Statistical Description !

What is Wave Chaos?



- Wave propagation- linear phenomena (response is linearly proportional to excitation)

- Therefore - not chaotic

- In complex geometries field distribution are highly sensitive to: frequency and/or small perturbations

- Classical rays are chaotic-this affects the field solutions

Two incident rays with slightly different initial directions have rapidly diverging trajectories

Random Matrix Theory and Wave Mechanics

- **Nuclear Spectra**

  Motivated by the then impossibility of calculating spectra of large nuclei, Wigner (1950's) conjectured that these spectra have statistical properties that are the same as those of suitable ensembles of random matrices.

- **Quantum Dots & Disordered Systems**

Y. Alhassid, Rev. Mod. Phys. **72**, 895 (2000).

C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731, (1997)

- **Acoustic Resonators**

R.L. Weaver, J. Acoust. Soc. Am. **85**, 1005 (1989)

- **Electromagnetic Compatibility**

L.K. Warne, K.S. H. Lee, H.G. Hudson, W. A. Johnson, R. E. Jorgenson, and S.L. Stronach, IEEE Trans. Ant. Prop. **51**, 978 (2003).

- **Quantum Chaos**

F. Haake, Quantum Signatures of Chaos (Springer-Verlag, Berlin 1991).

H.-J. Stöckmann, Quantum Chaos (Cambridge University Press, Cambridge 1999)

Eigenfrequency Statistics

$$\underline{\underline{H}} \cdot \underline{\psi}_n = E_n \underline{\psi}_n$$

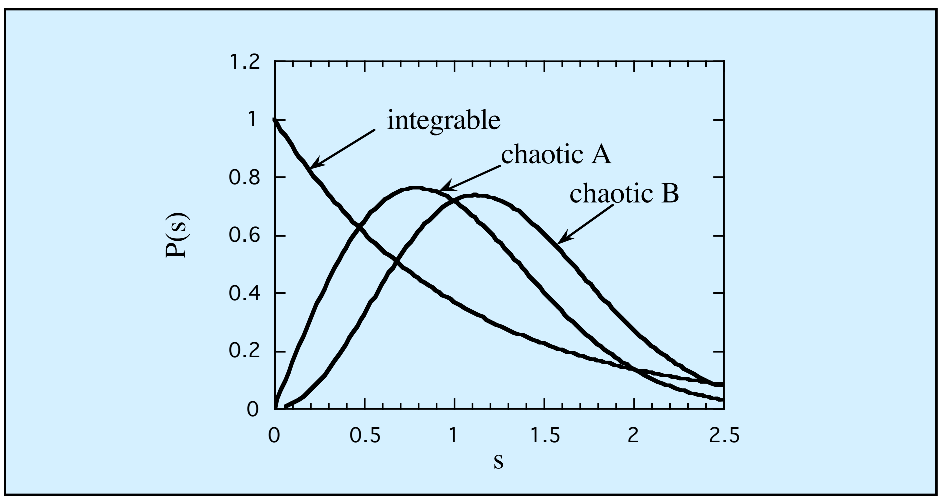
Eigenvalues E_n are distributed particular statistics

Mean Spacing: $\Delta E = \langle E_{n+1} - E_n \rangle$

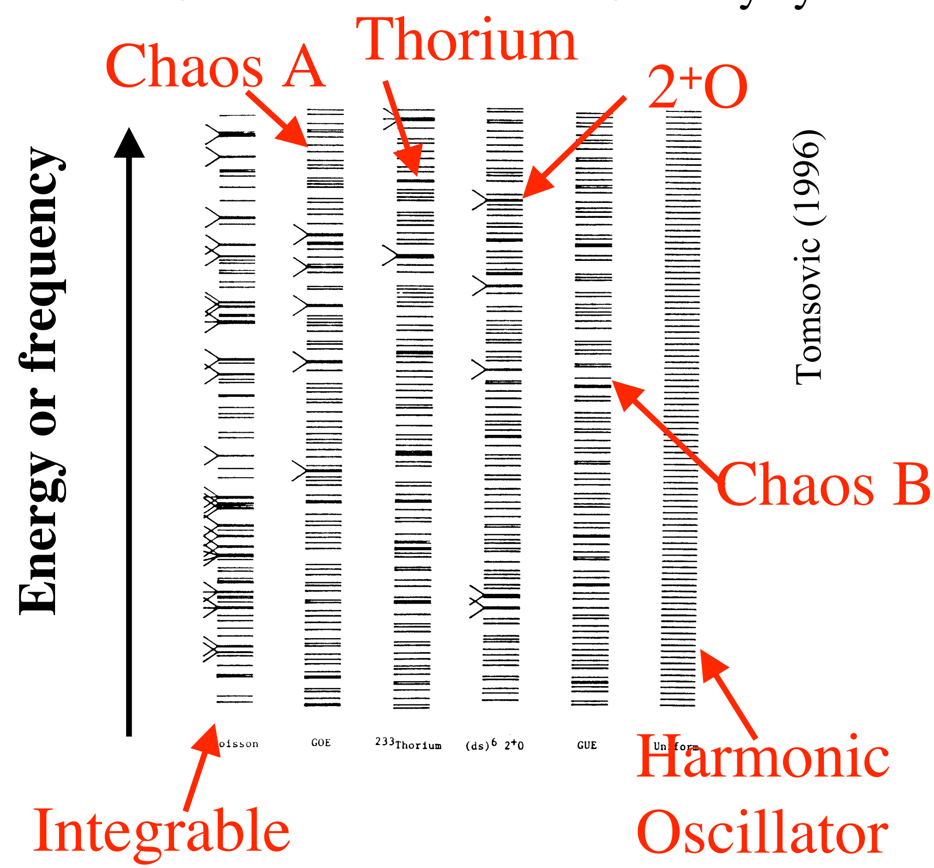
Normalized Spacing:

$$s_n = \langle E_{n+1} - E_n \rangle / \Delta E$$

Distribution



Spacing distributions are characteristic for many systems

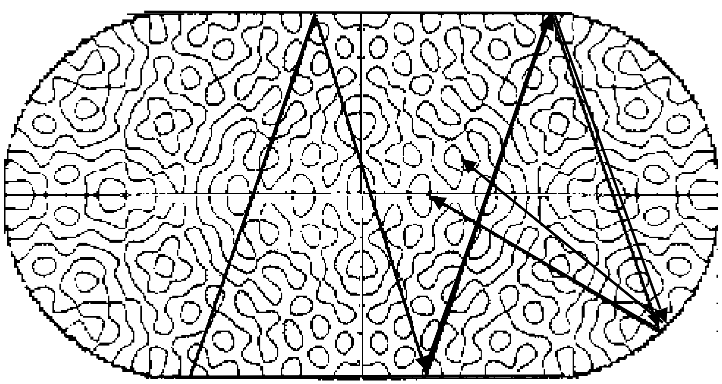


Chaos A = Time reversal symmetry

Chaos B = Time reversal symmetry broken

RMT and Eigenfunction Statistics

Hypothesis: Random matrix theory (RMT) applies to wave problems in the semiclassical regime (short wavelength), if the ray approximation corresponding to the given wave problem yields chaotic wave trajectories [McDonald and Kaufman, PRL (1979); Bohigas, et al., PRL (1984)].



Replace eigenfunction with superposition of random plane waves (Berry Hypothesis)

$$\phi_n = \lim_{N\to\infty} \mathrm{Re}\left\{\sqrt{\frac{2}{AN}}\sum_{k=1}^{N} a_k \exp\left[i\left(k_n \boldsymbol{e}_k \cdot \boldsymbol{x} + \theta_k\right)\right]\right\}$$

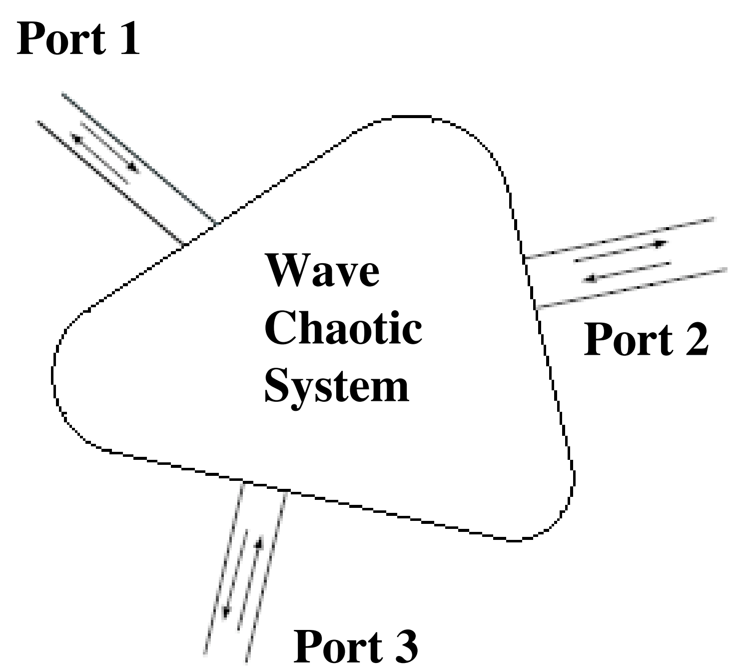
Random amplitude

Random direction

Random phase

Note: Hypothesis now assumed to apply even for simple systems.

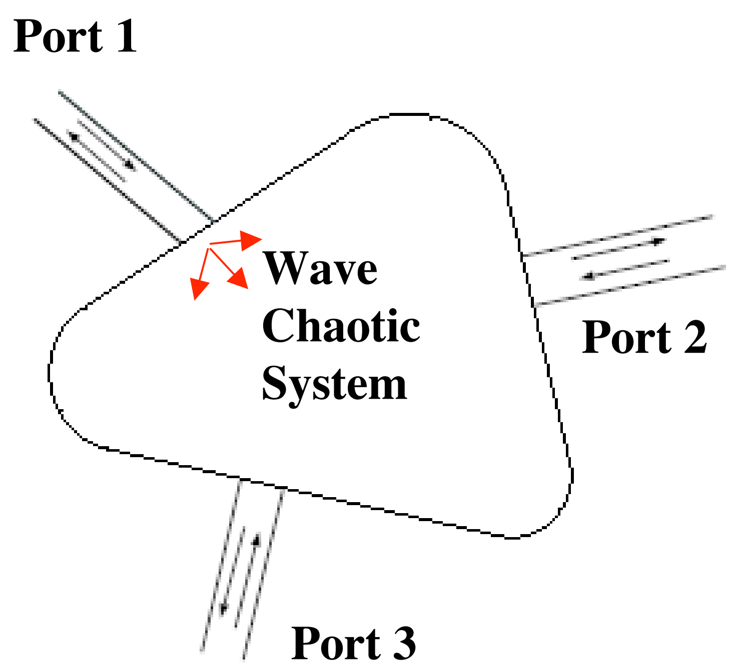
Our Problem - Scattering



Q: What is the nature of the interaction of the wave chaotic system with the outside world via the connecting ports?

A difficulty: While the waves within the chaotic system are presumably described by random matrix theory in a 'universal' (system independent) manner, the answer to Q. also depends on the non-universal aspects of the specific geometry of the coupling between the ports and the chaotic system.

Universal and System Specific Aspects

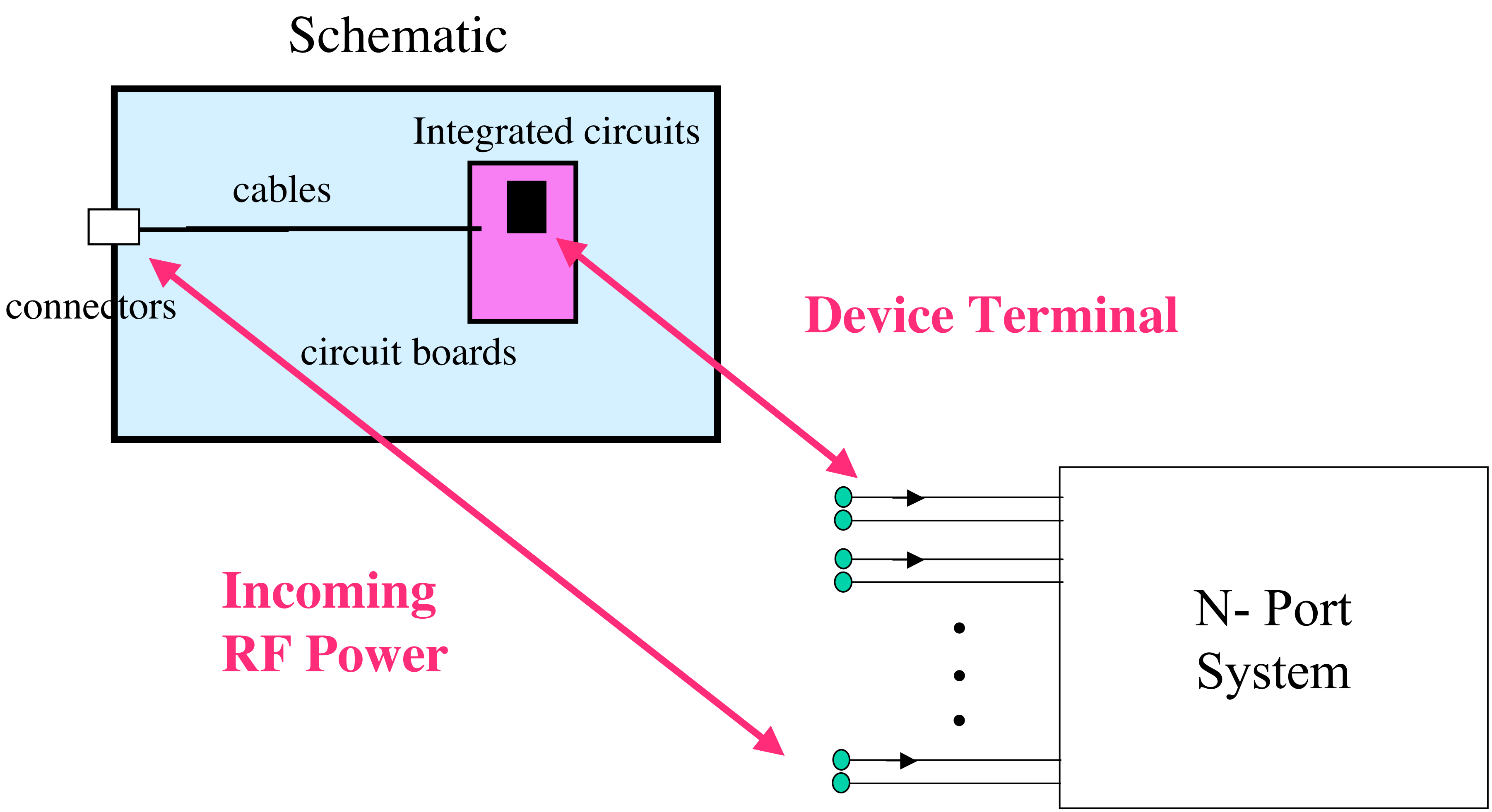


1. **Entry of energy to cavity determined by properties of port.**
   - System specific
   - Applies also to exit

2. **Wave fields inside described by RMT and Berry Hypothesis**
   - Universal statistics
   - Some system specific dependence

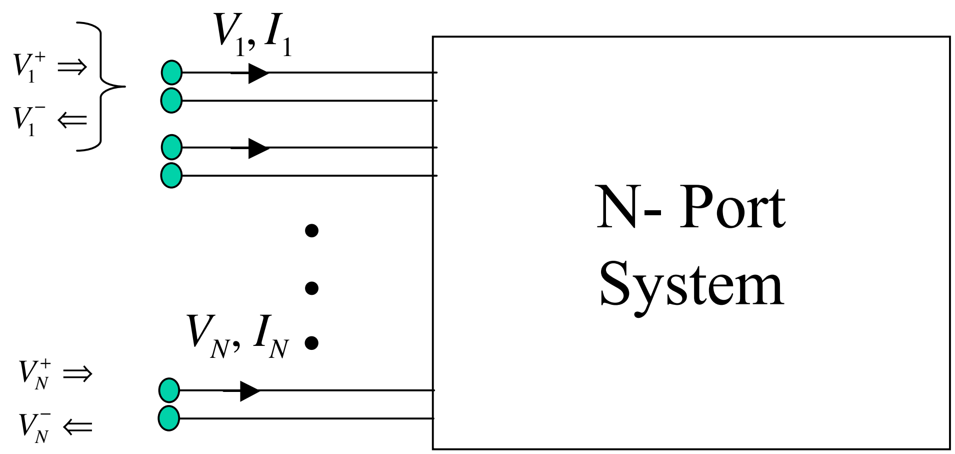
Note, interference is still accounted for.

Black Box Representation
N - Port System



Z and S-Matrices
What is S_{ij} ?

N ports
• voltages and currents,
• incoming and outgoing waves



Z matrix

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} = \boldsymbol{Z} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_2 \end{pmatrix}$$

voltage current

S matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_{N1}^- \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_{N1}^+ \end{pmatrix}$$

outgoing incoming

$$\boldsymbol{S} = (\boldsymbol{Z} + \boldsymbol{Z}_0)^{-1}(\boldsymbol{Z} - \boldsymbol{Z}_0)$$

$$\boldsymbol{Z(\omega)}\ ,\ \boldsymbol{S(\omega)}$$

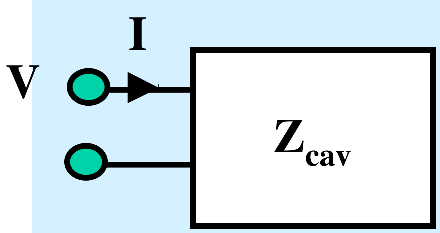
• Complicated function of frequency
• Details depend sensitively on unknown parameters

Frequency Dependence of Reactance for a Single Realization

Lossless:

$V = Z_{cav}\, I$

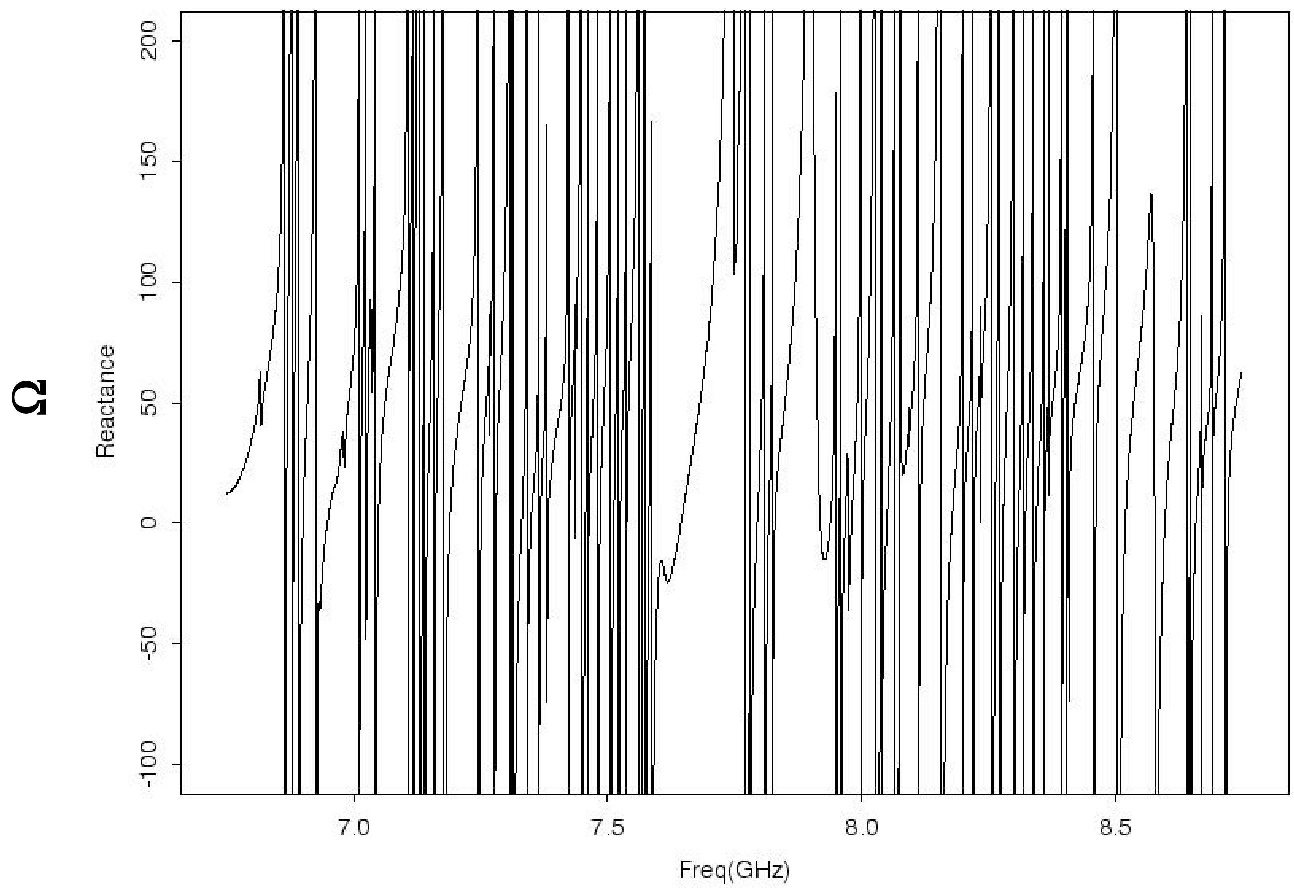
$Z_{cav} = jX_{cav}$



S - **reflection coefficient**

$$S = \frac{Z_{cav} - Z_0}{Z_{cav} + Z_0} = e^{j\phi}$$

Mean spacing δf ≈ .016 GHz



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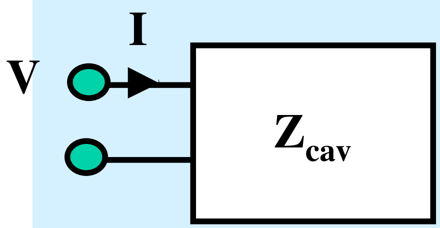

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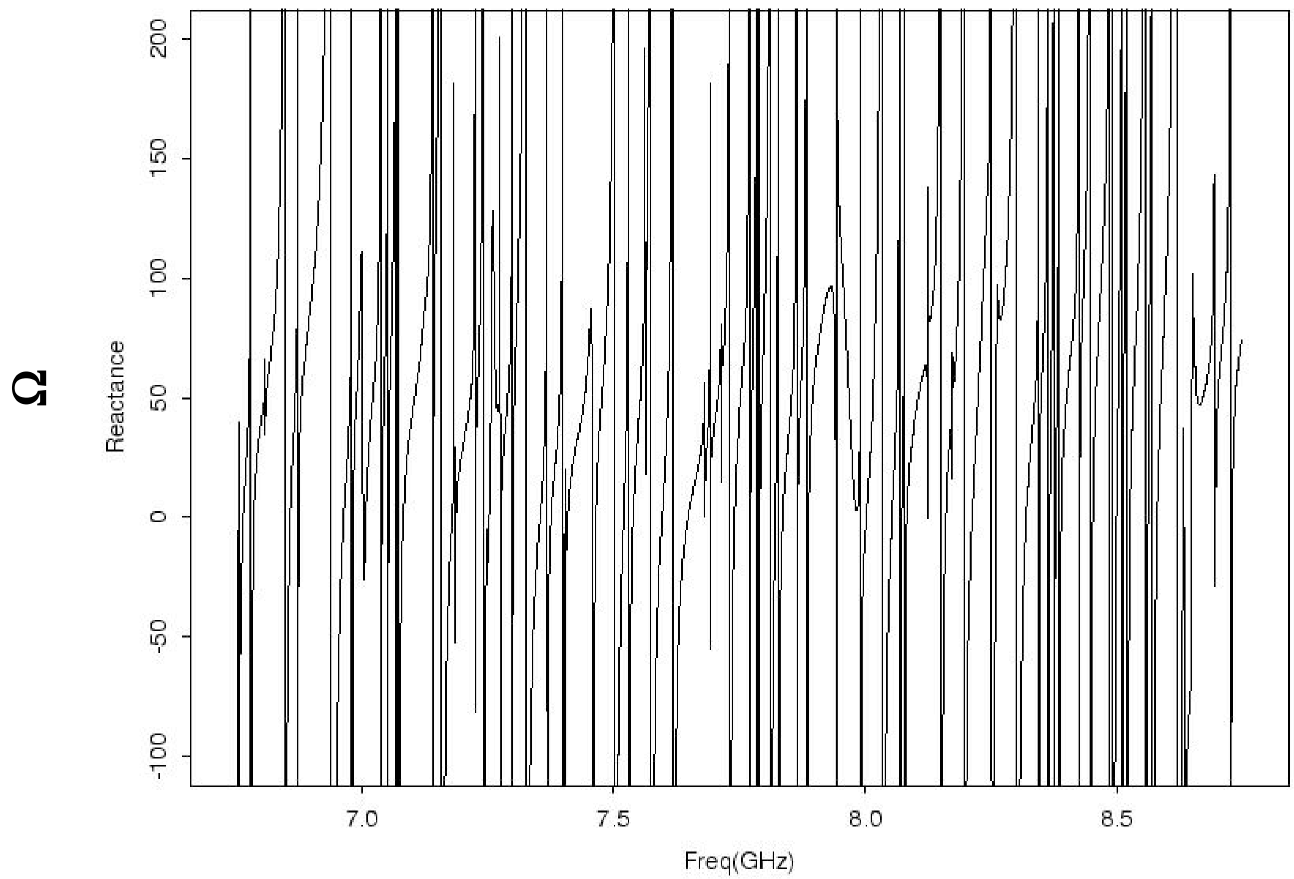
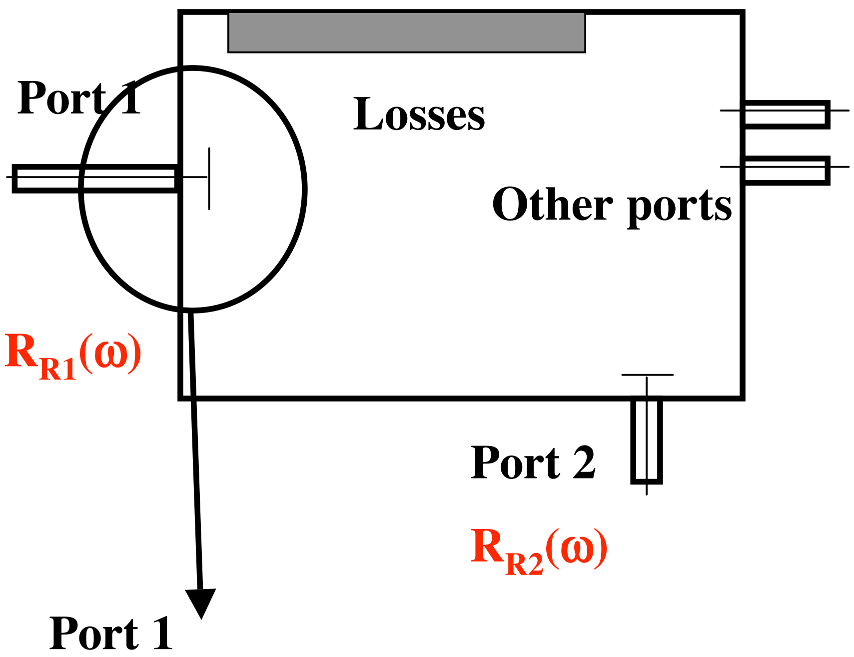
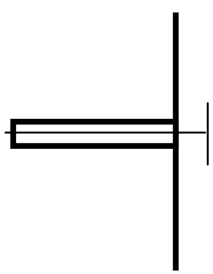


Statistical Model of Z Matrix





Free-space radiation Resistance $R_R(\omega)$

$Z_R(\omega) = R_R(\omega) + jX_R(\omega)$

Statistical Model Impedance

$$Z_{ij}(\omega) = -\frac{j}{\pi}\sum_n R_{Ri}^{1/2}(\omega_n) R_{Rj}^{1/2}(\omega_n) \frac{\Delta\omega_n^2 \, w_{in} w_{in}}{\omega^2(1 + jQ^{-1}) - \omega_n^2}$$

System parameters
- **Radiation Resistance** $R_{Ri}(\omega)$
- $\Delta\omega^2_n$ **- mean spectral spacing**
- Q **-quality factor**

Statistical parameters
- ω_n **- random spectrum**
- w_{in}**- Guassian Random variables**

Compact Expression for Impedance Matrix

Statistical Model for Cavity Impedance

$$\underline{\underline{Z}}^{cav} = i\,\mathrm{Im}\left(\underline{\underline{Z}}^{rad}\right) + \left[\underline{\underline{R}}^{rad}\right]^{1/2} \cdot \underline{\underline{\xi}} \cdot \left[\underline{\underline{R}}^{rad}\right]^{1/2}$$

Radiation Impedance - Ports

$$\underline{\underline{Z}}^{rad} = i\,\mathrm{Im}\left(\underline{\underline{Z}}^{rad}\right) + \underline{\underline{R}}^{rad}$$

Statistics generated by RMT

$$\underline{\underline{\xi}} = \frac{i}{\pi}\sum_n \frac{\Delta k^2\ \underline{w}_n \underline{\tilde{w}}_n}{\left[k_0^2(1+i/Q) - k_n^2\right]}$$

GRVs

Eigenvalues of a RM

Lorentzian Random Matrix ($\alpha = k_o^2/(Q\Delta k^2)$)

Two Approaches

- $Z(\omega)$ construction - Microscopic approach

  Uses pieces of problem: R_R, $\Delta\omega^2$, Q to construct Z and S.

- Poisson Kernel - Global Approach to S

(P. A. Mello, P. Pereyra, and A. Frankel, Ann of Phys. 161, 251 (1985))

  Statistics of S - Matrix determined by its average E{S}

  - P(S| E{S}) includes details of coupling

- Equivalence: Brouwer, Phys. Rev B51, 16878 (1995)

Some Relevant References

P. A. Mello et al., Ann. Phys. **161**, 254 (1985).

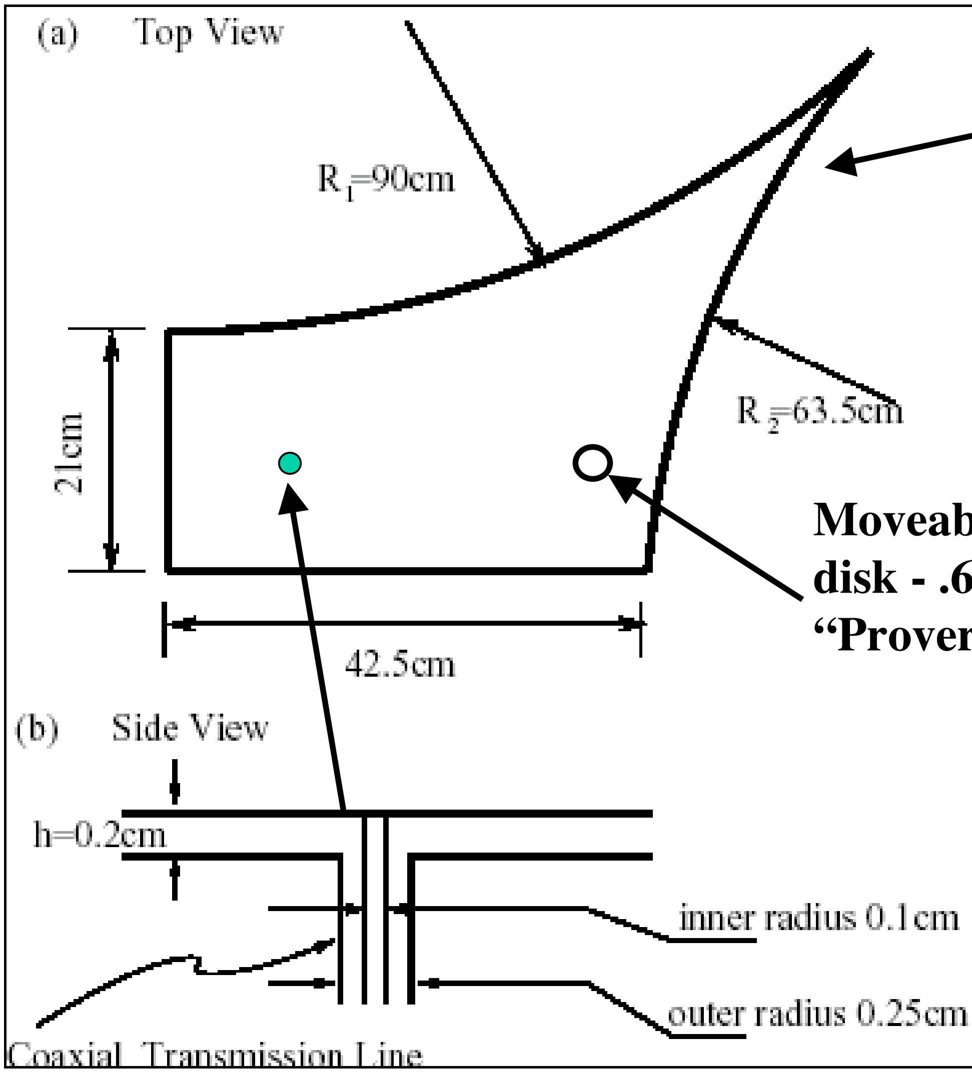
P. W. Brouwer, Phys. Rev. B **51** (1995).

E. Kogan et al., Phys. Rev. E **61** (2000).

R. A. Mendez-Sanchez et al., Phys. Rev. Lett. **91**, (2003).

L. K. Warne et al., IEEE Trans. Antennas & Prop. **51**, 978 (2003).

HFSS - Solutions
Bow-Tie Cavity



Expect: $Z = j(X_R + R_R \xi)$

ξ - unit Lorentzian

Krieger, Ann. Phys. **42**, 375 (1967)
Mello, "Mesoscopic Quantum Physics" 1995
Fyodorov and Sommers, J. Math Phys. **38**, 1918 (1997)

Moveable conducting disk - .6 cm diameter "Proverbial soda can"

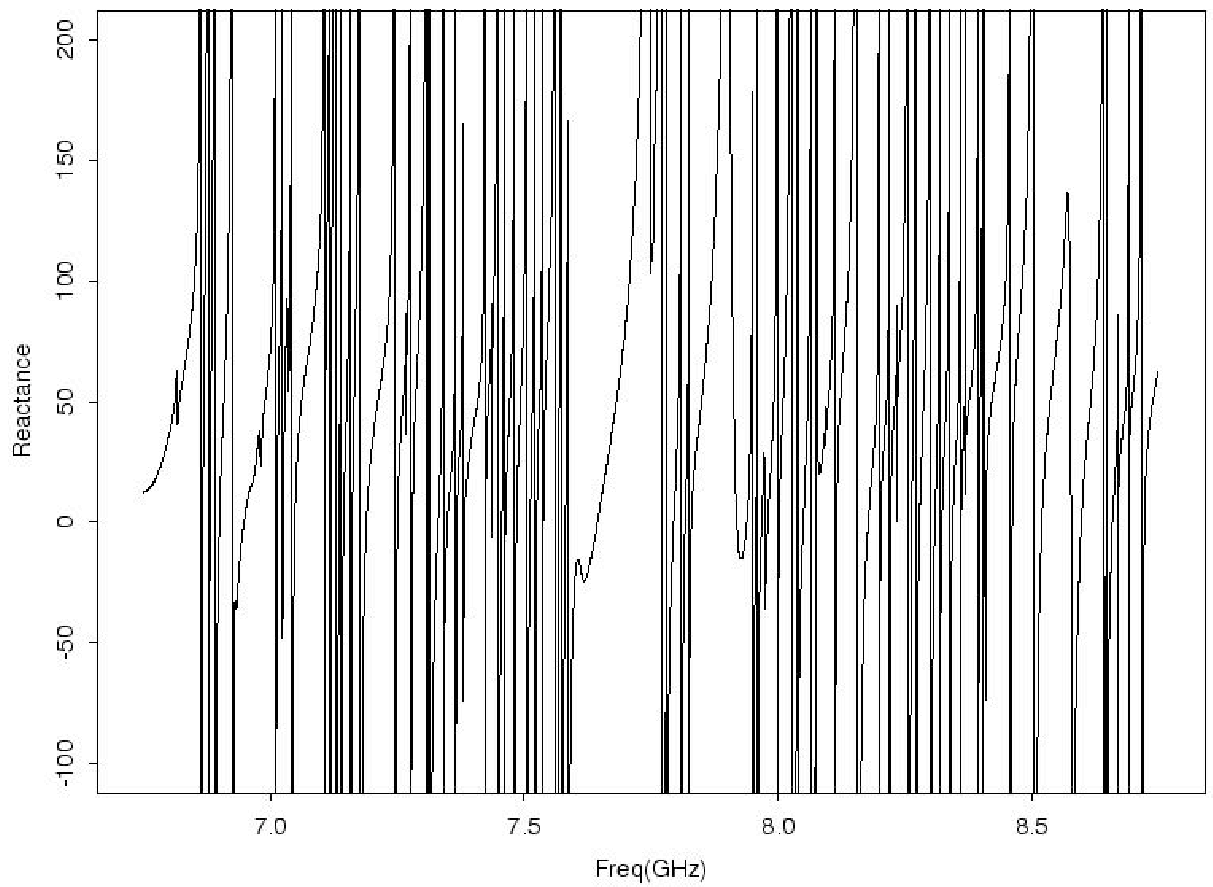
Cavity impedance calculated for
100 locations of disk
4000 frequencies
6.75 GHz to 8.75 GHz

Frequency Dependence of Reactance for a Single Realization

Mean spacing $\delta f \approx .016$ GHz

Ω

$$Z_{cav} = jX_{cav}$$

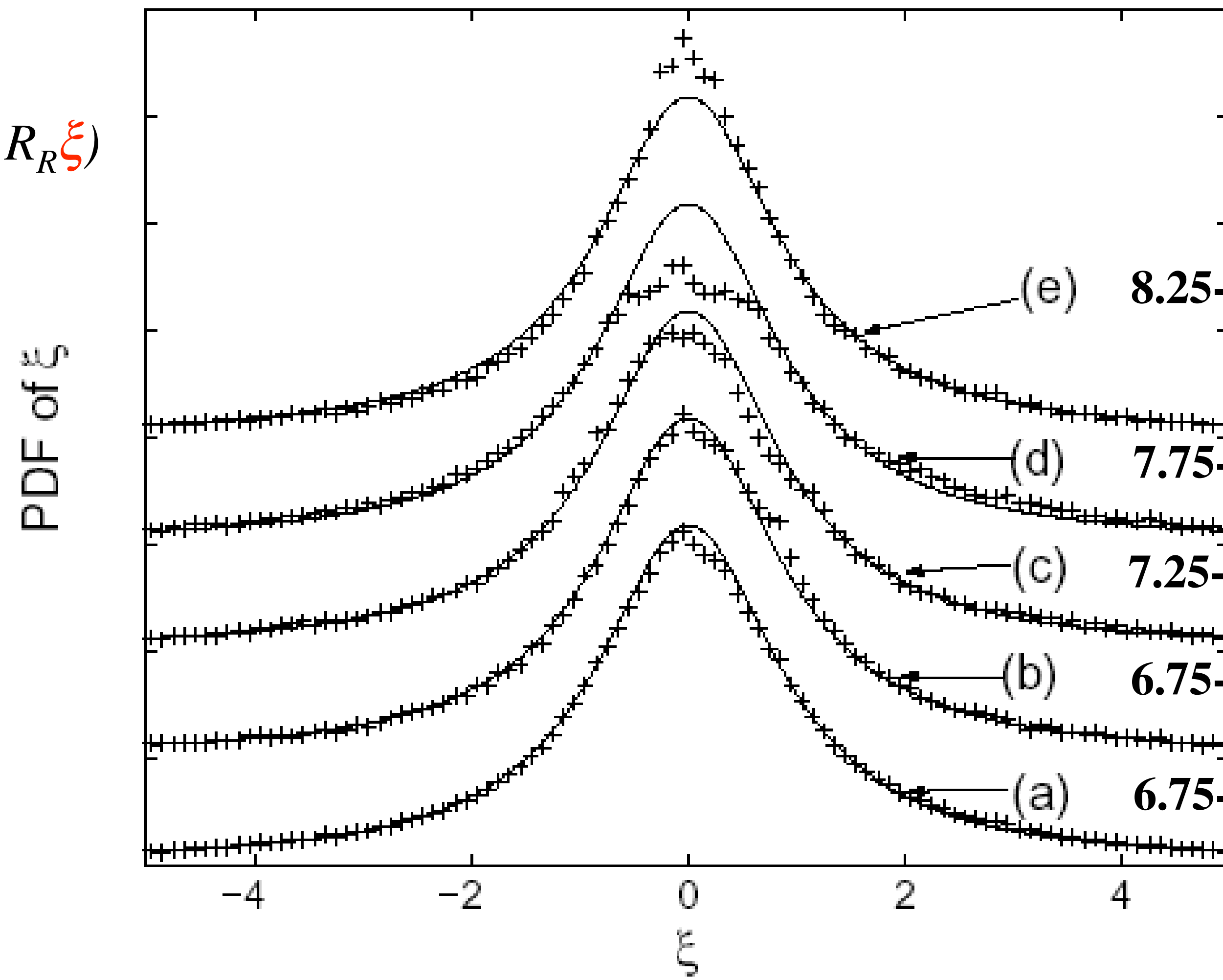


Distribution of Fluctuating Cavity Impedance

$$\xi = (X - X_R(\omega)) / R_R(\omega)$$

$Z = j(X_R + R_R\xi)$

$\boldsymbol{R_R \approx 35\ \Omega}$



8.25-8.75 GHz

7.75-8.25 GHz

7.25-7.75 GHz

6.75-7.25 GHz

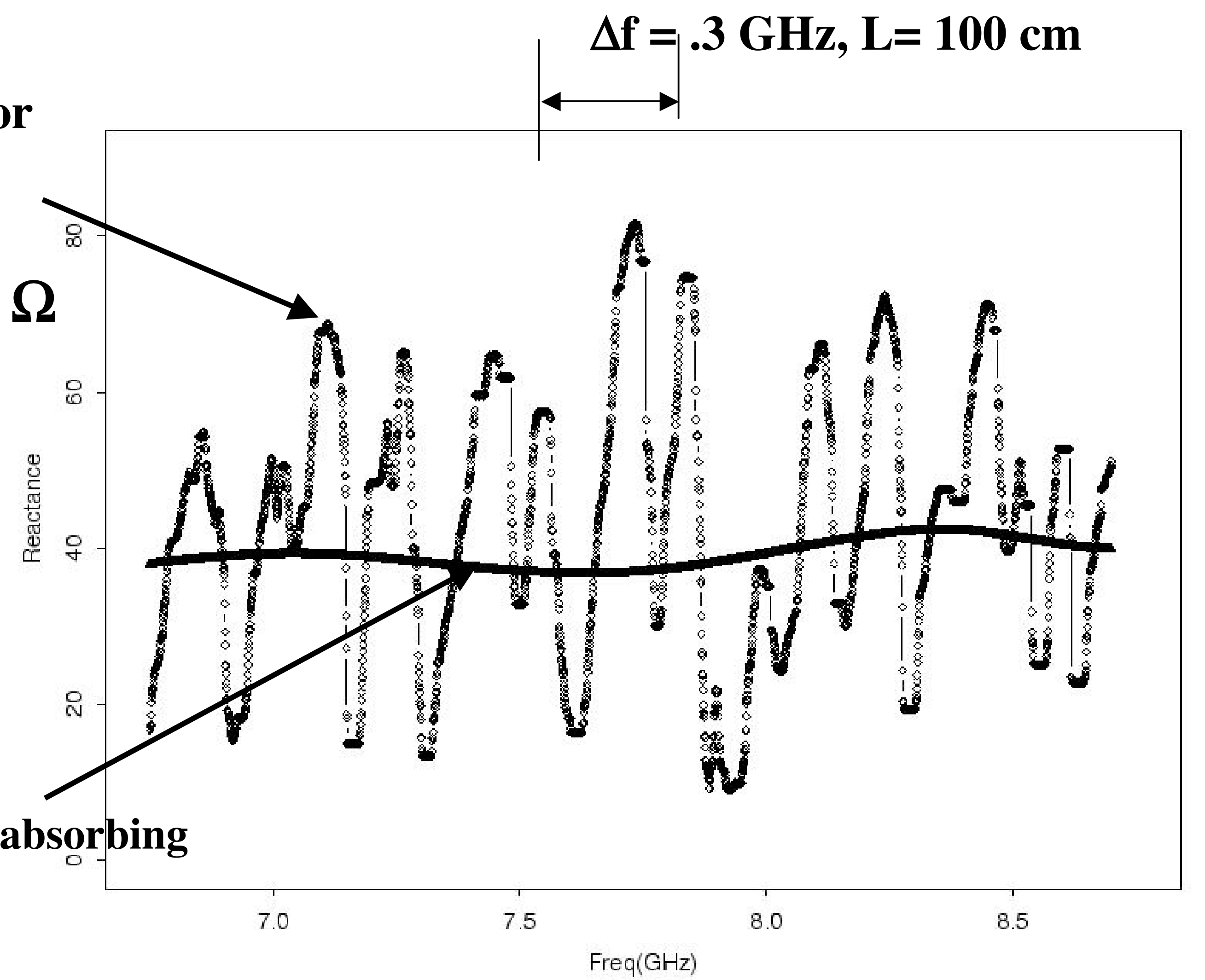
6.75-8.75 GHz

Frequency Dependence of Median Cavity Reactance

Median Impedance for 100 locations of disc

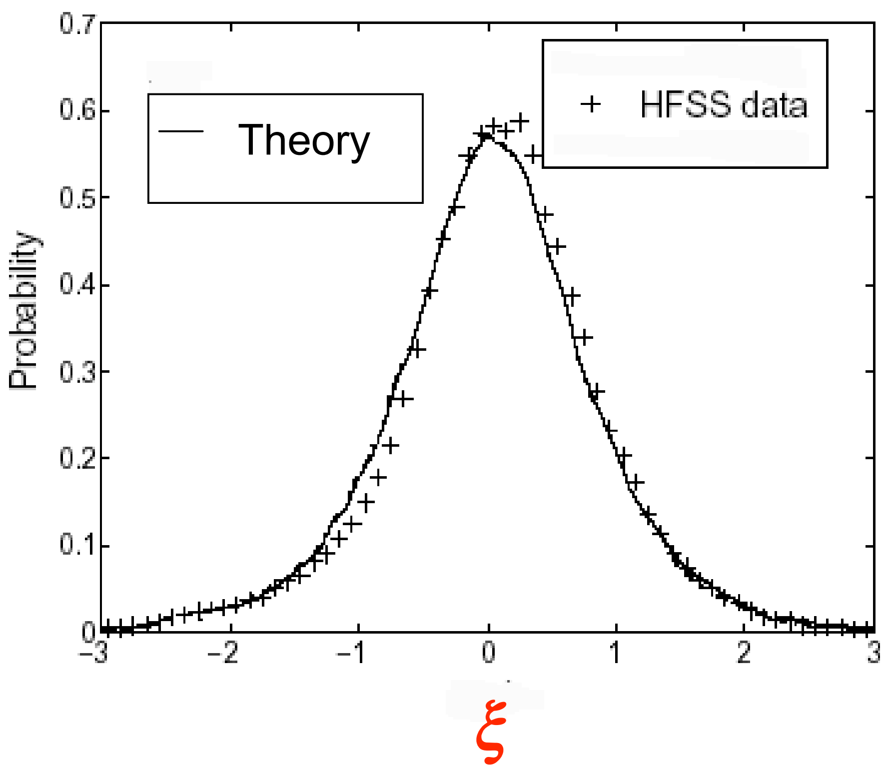
Effect of strong Reflections ?

Radiation Reactance HFSS with perfectly absorbing Boundary conditions

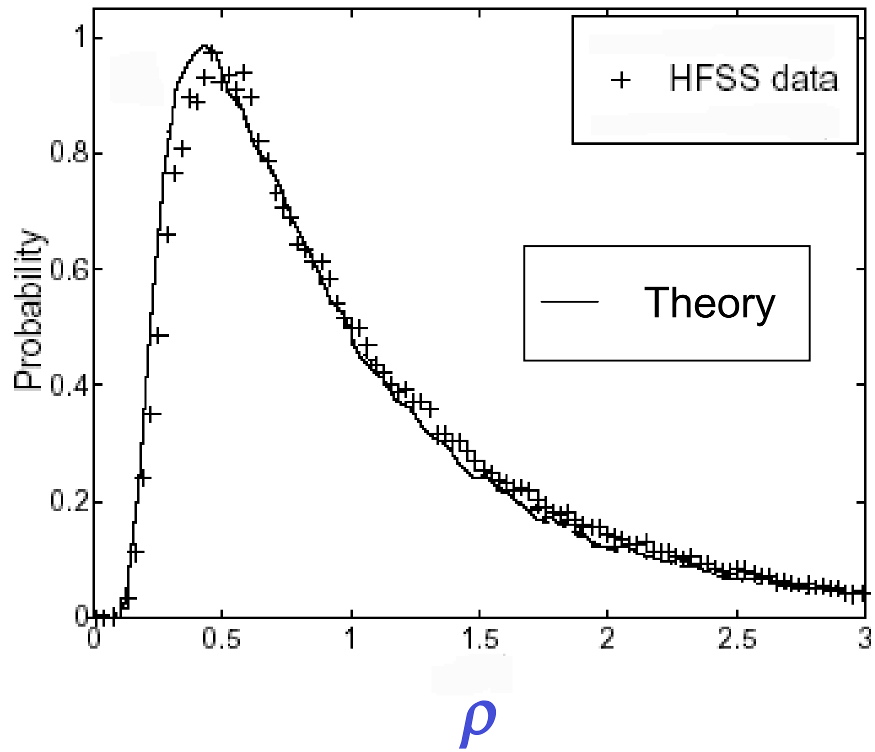


Comparison of HFSS Results and Model for Pdf's of Normalized Impedance

Normalized Reactance



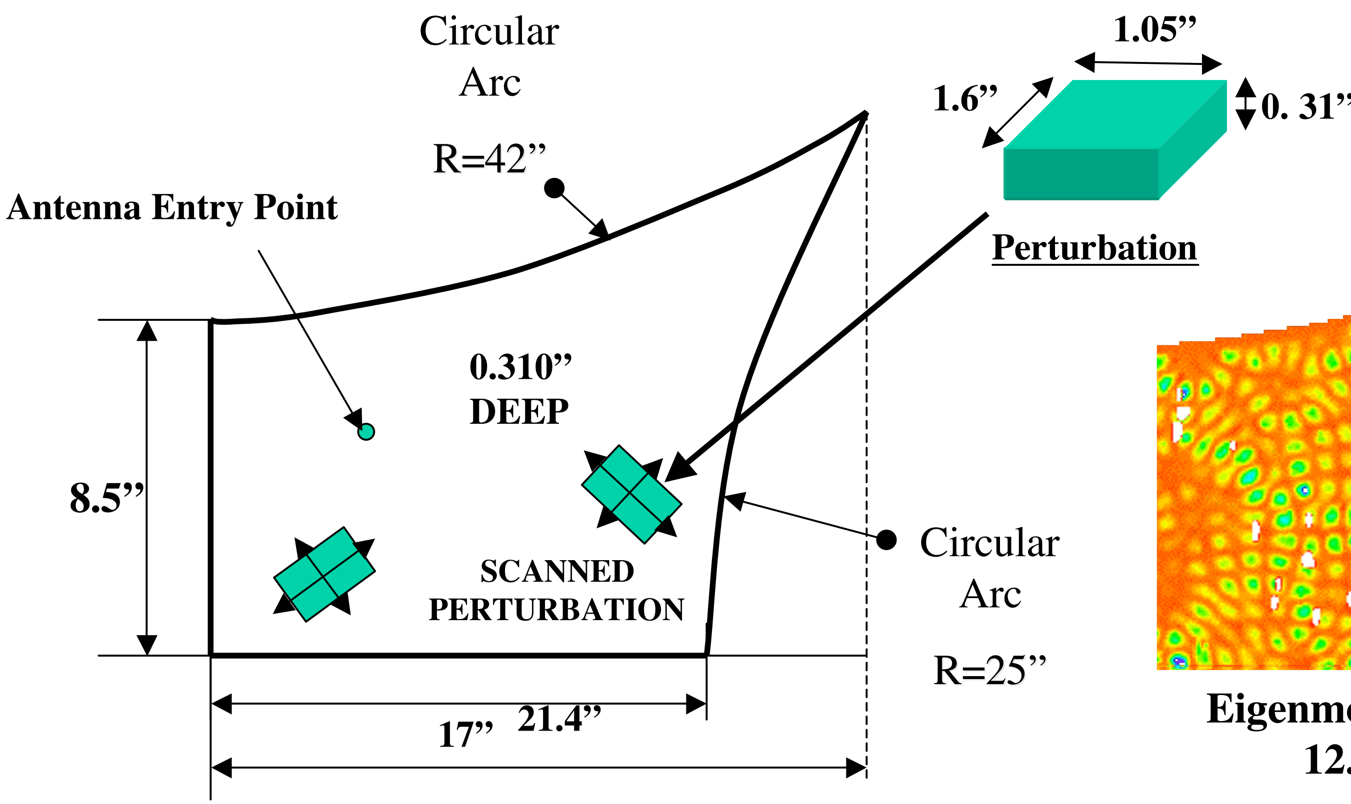
Normalized Resistance

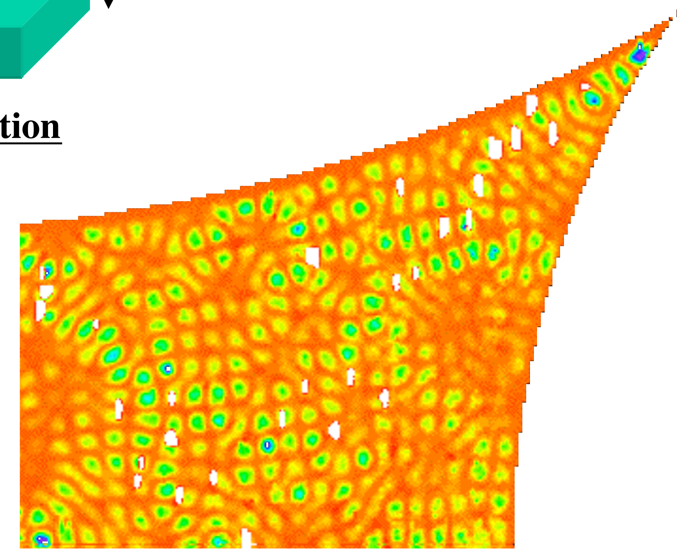


$$Z_{cav} = jX_R + (\rho + j\xi)\, R_R$$

EXPERIMENTAL Test

S. Hemmady et al., PRL 94, 014102 (2005)



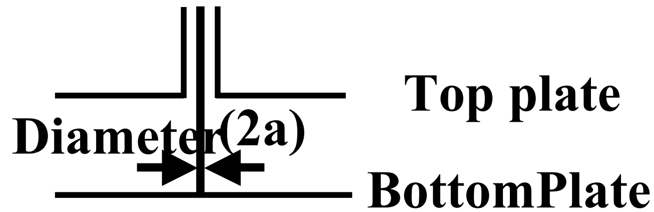


Eigenmode Image at 12.57GHz

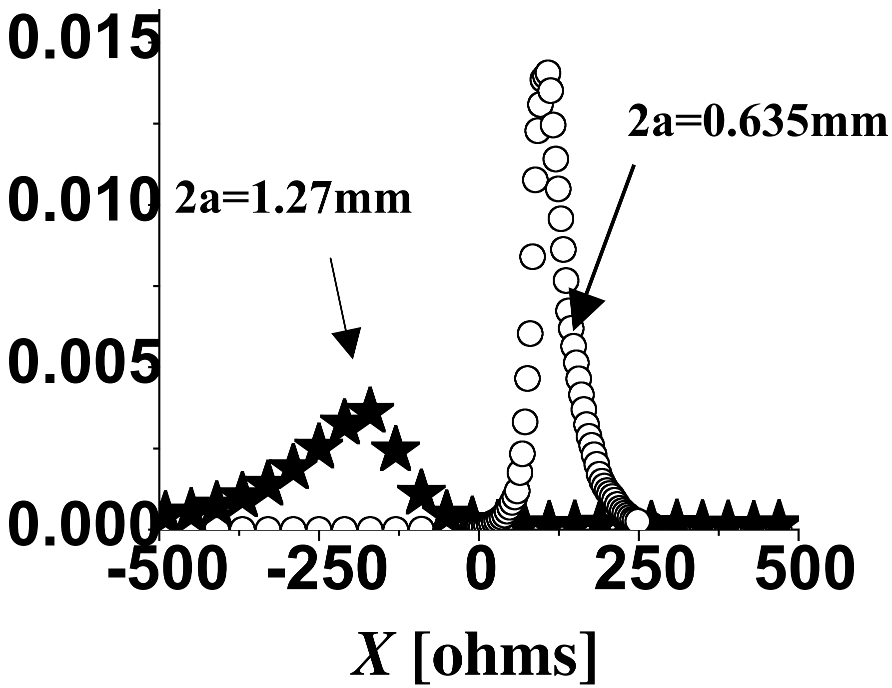
- 2 Dimensional Quarter Bow Tie cavity
- 1-port S and Z measurements in the 6 – 12 GHz range.
- Ensemble average through 100 locations of the perturbation

Importance of Normalization

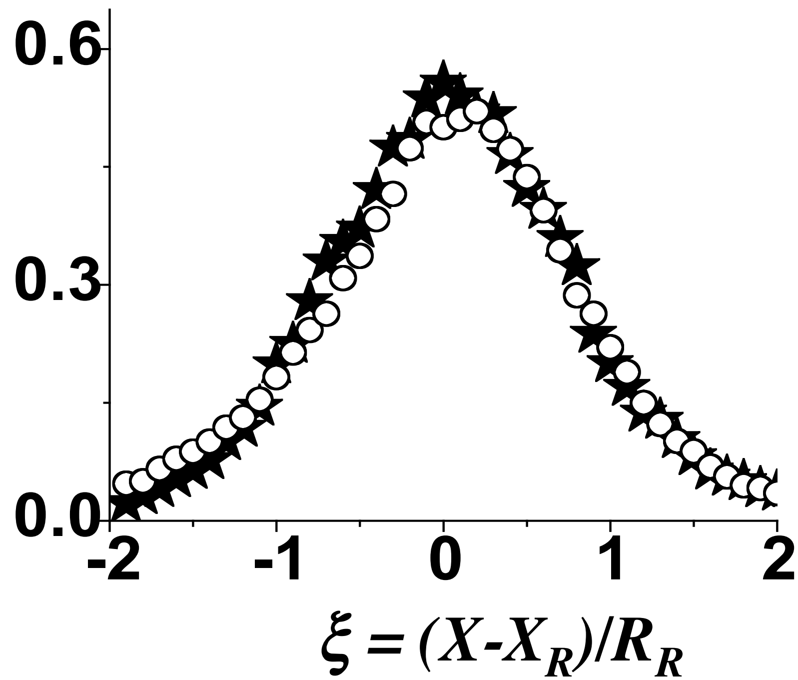
$Z = j(X_R + R_R\xi)$



Raw Data

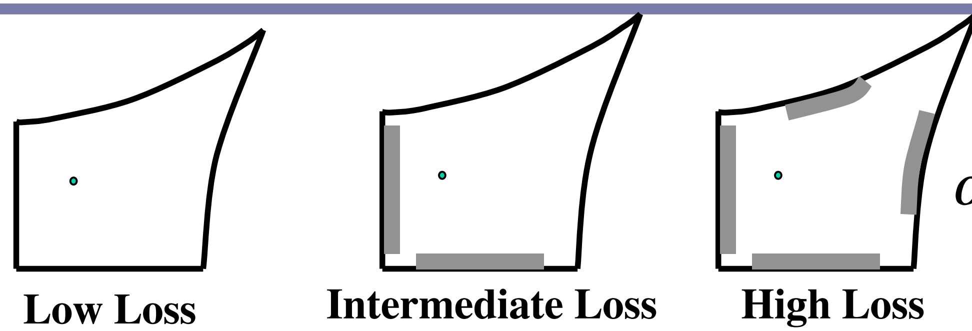


Normalized

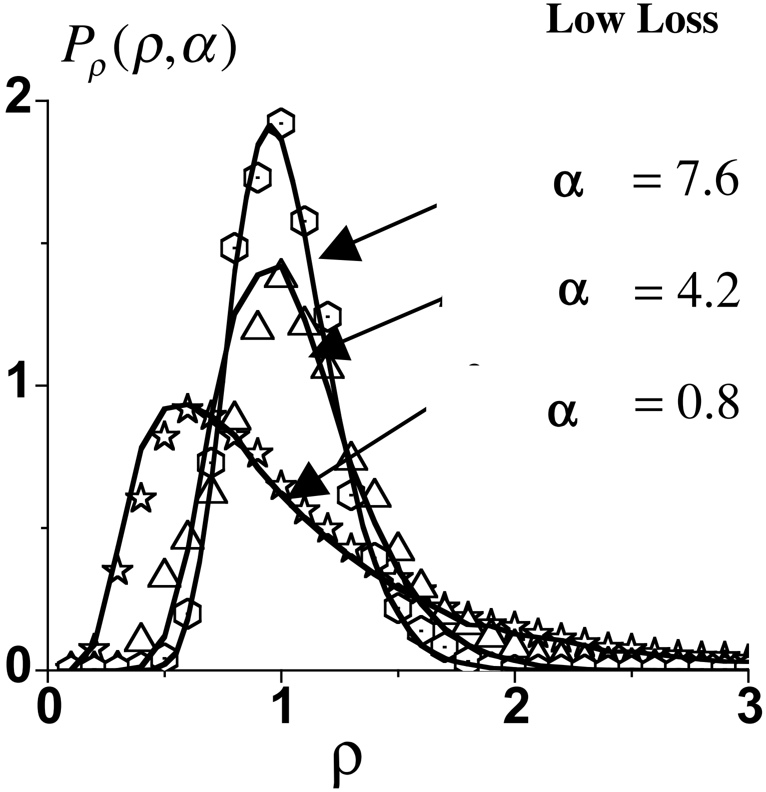


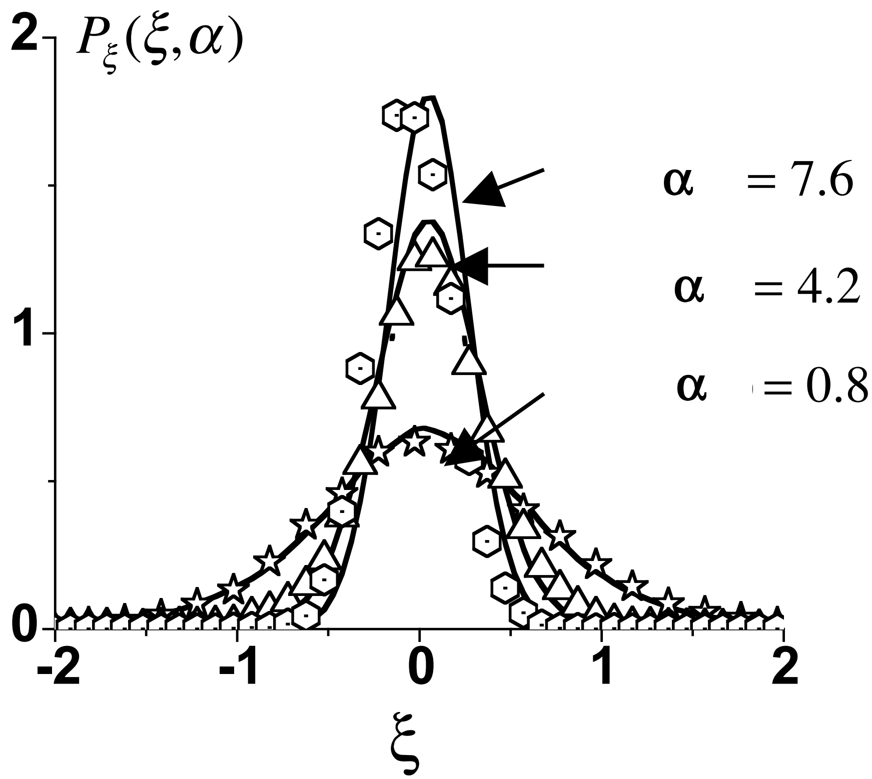
Testing the Effects of Varying Loss

$$Z = jX_R + R_R(\rho + j\xi)$$



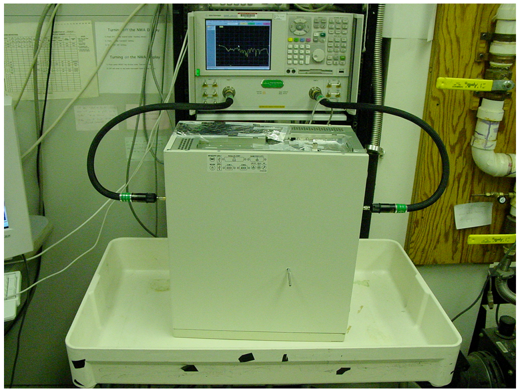
$$\alpha = \frac{\omega}{2Q\Delta\omega}$$



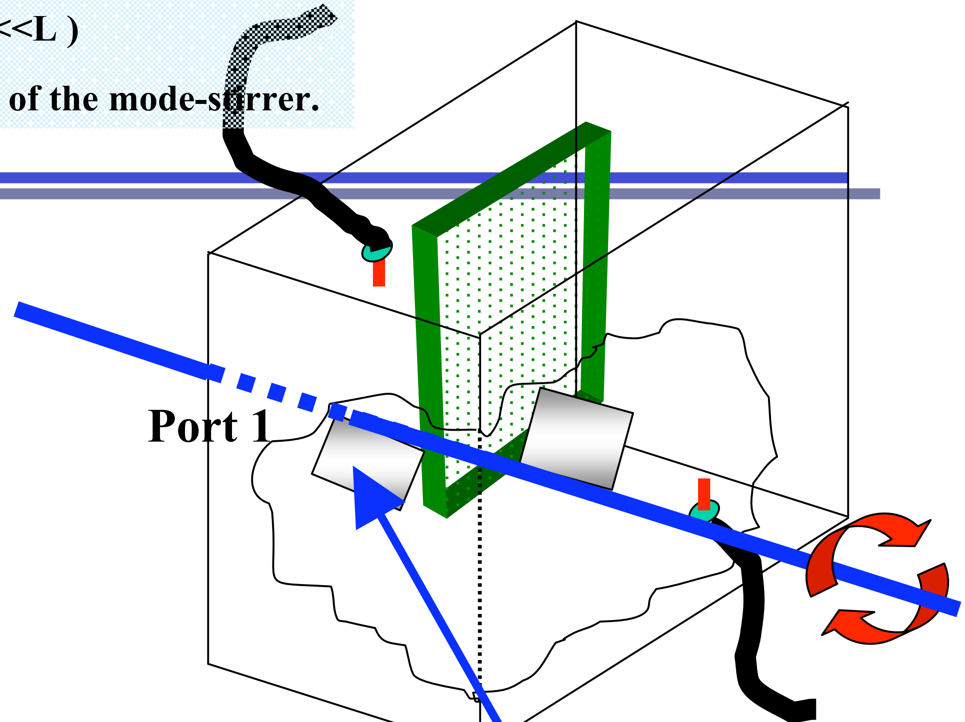


Validation of the Model 3D - S. Hemmady

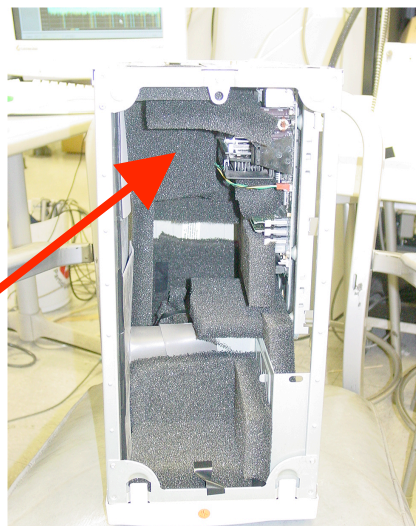
- Frequency Range: 2GHz to 20 GHz (λ<<L)
- Ensemble Averaging over ~20 positions of the mode-stirrer.



Experimental Setup [Cavity Case]



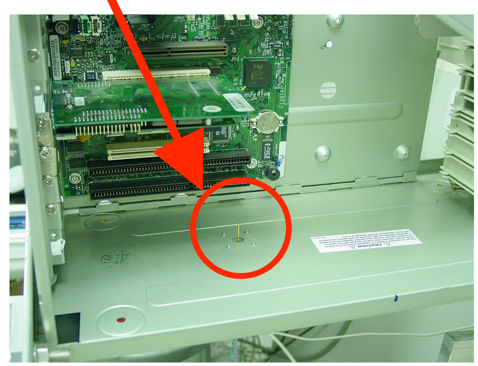
Paddle-Wheel Mode-Stirrer

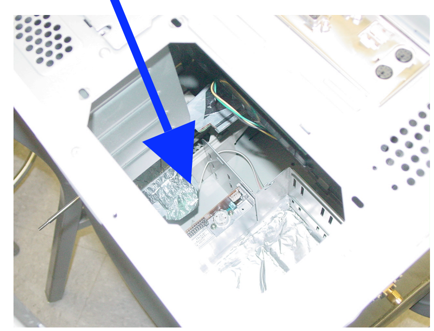


Microwave absorber

Port Radiation Measurement Setup

Port 2

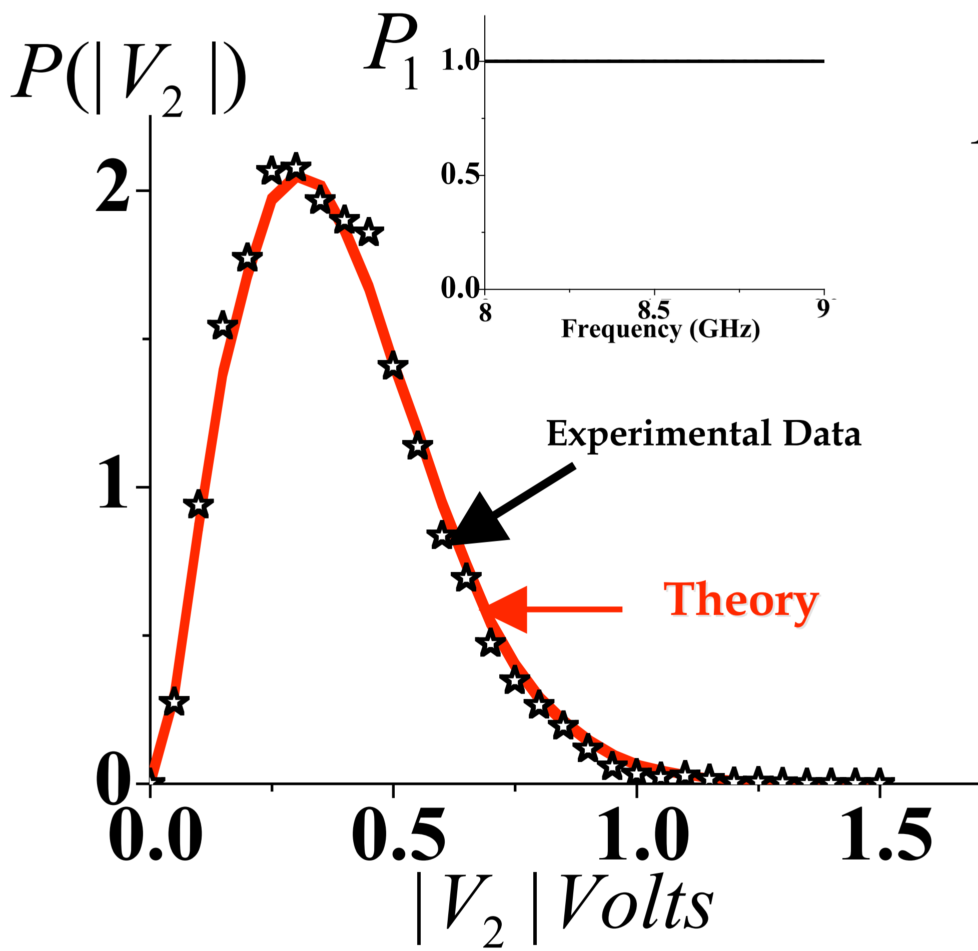


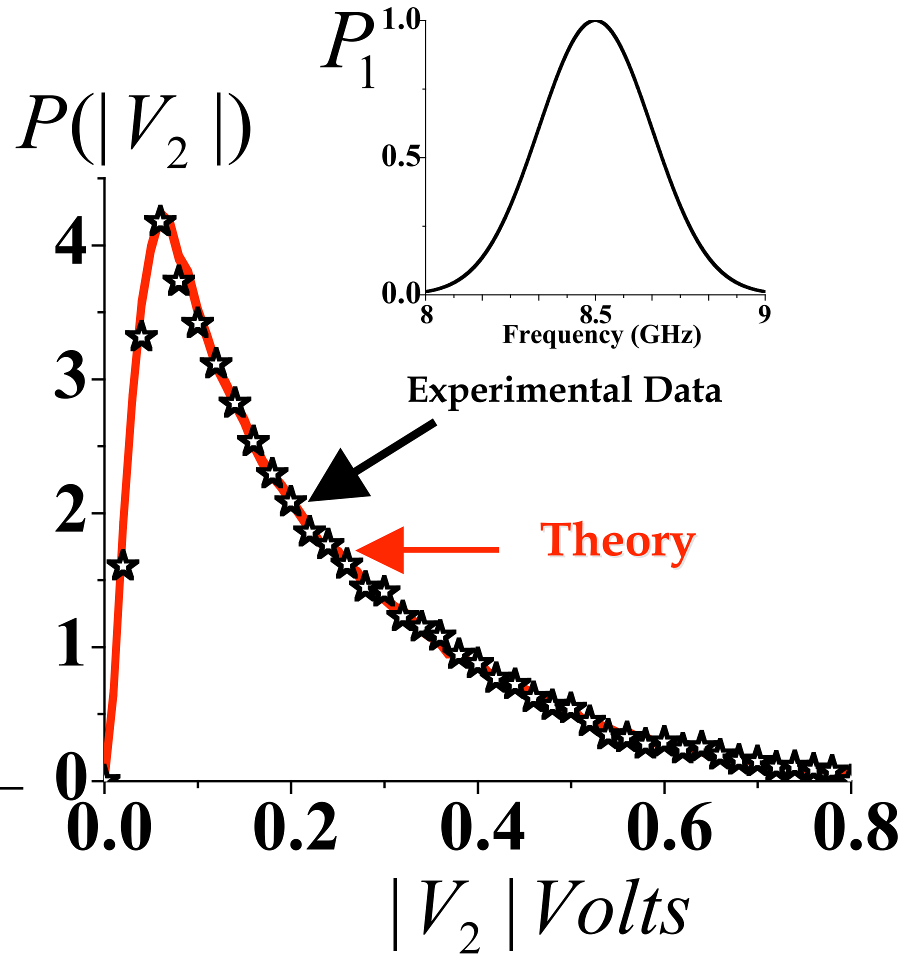


PDF of voltages on port 2 for different power-spectral densities radiated at Port 1

Flat PSD radiated from port 1

Gaussian-shaped PSD radiated from port 1





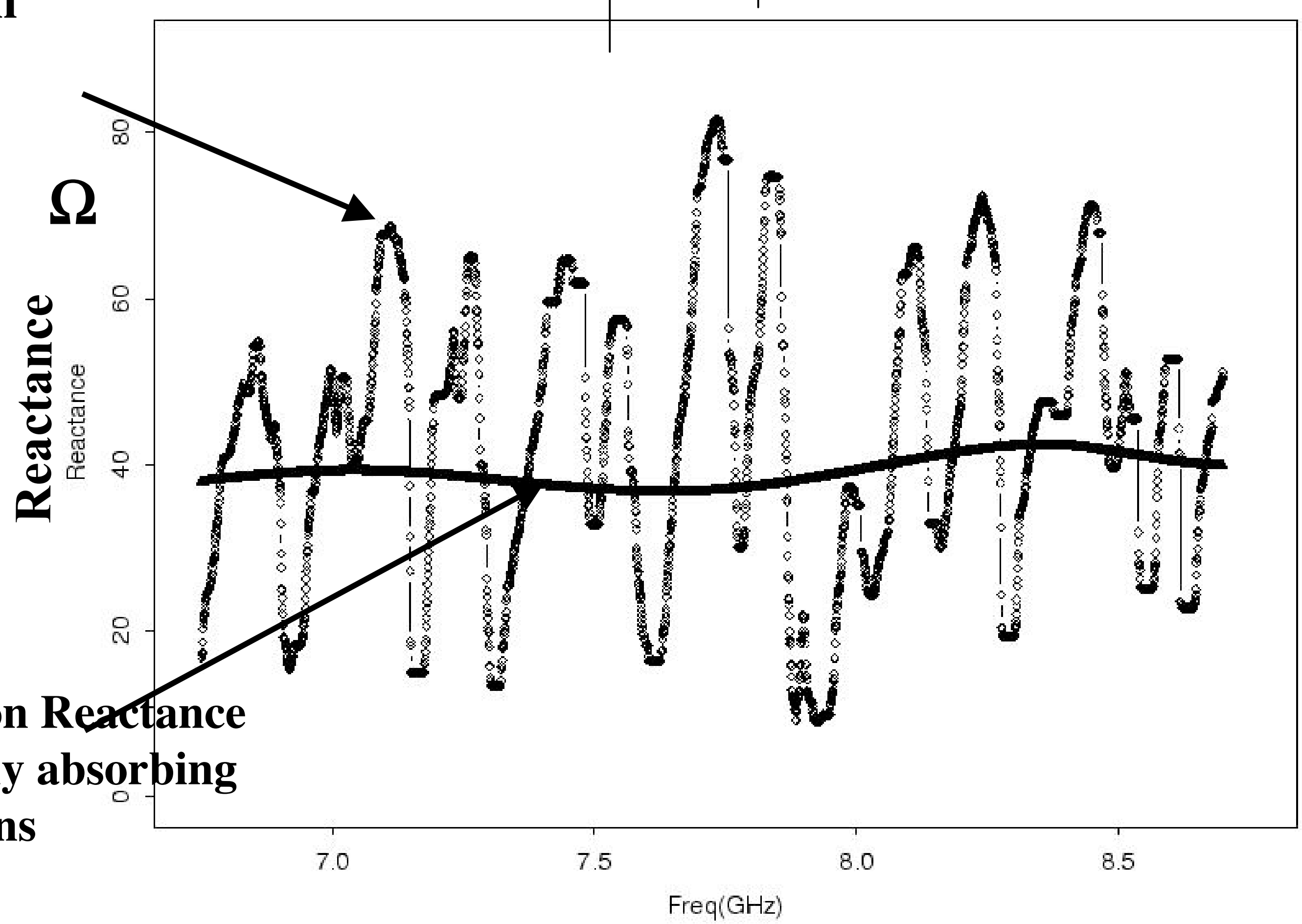
Unexpected Frequency Dependence of Median Cavity Reactance

Δf_{res} = .016 GHz

Δf = .3 GHz, L= 100 cm

Calculated: Median Reactance for100 locations of disc

Effect of Reflections ?



Expected: Radiation Reactance HFSS with perfectly absorbing Boundary conditions

Effect of Direct Ray Paths

Original Random Coupling Model (RCM)

- **RF energy is randomized on entering cavity**
- **Only radiation impedance of ports, cavity volume and average Q are important**

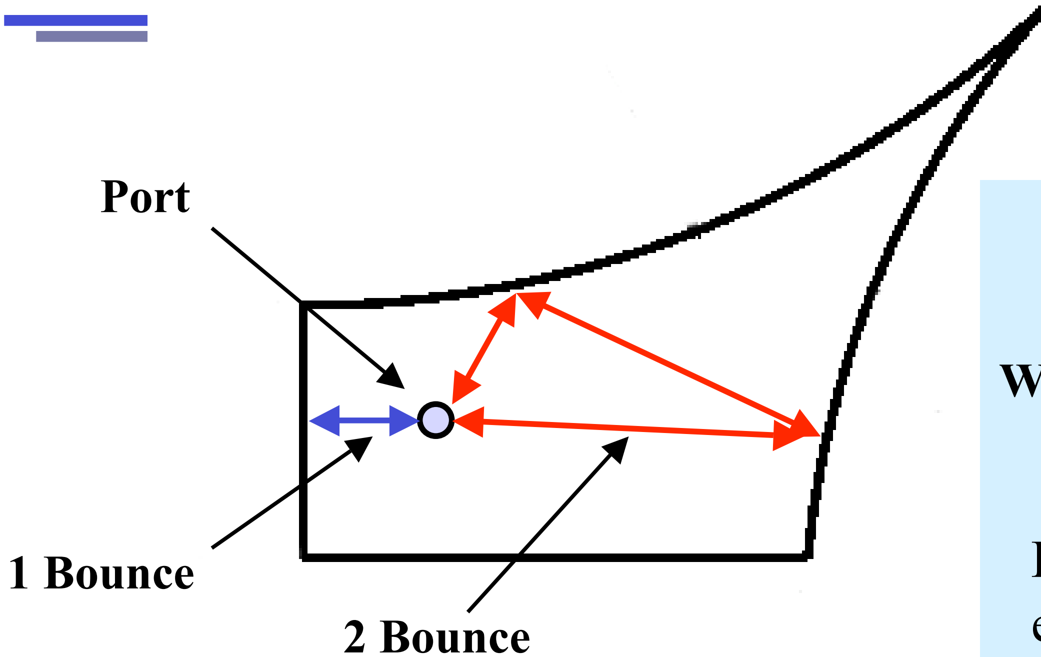
In some geometries, or in narrow frequency bands specifics of internal geometry are important

Modified Random Coupling Model

- **J. Hart et al.,** PHYSICAL REVIEW E 80, 041109 (2009)
- **Allows for systematic improvement by inclusion of geometric details if known**
- **Can be used in conjunction with measured data**

Using the Random Coupling Model, extended to include the effect of direct paths, we can now derive these two fading distributions.

Effect of Short Ray Paths
Hart et al. PRE 80, 041109 (2009).



etc.

$$Z = j(X_{R,N}+R_{R,N}\xi)$$

Where:

$$Z_{R,N} = R_{R,N} + j\,X_{R,N}$$

Radiation impedance including the effect of N-bounces off the wall calculated in the semi-classical approximation

Semi-classical Green's function in 2D

$$G(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{4}\left[H_0^{(1)}\left(k\left|\boldsymbol{r}-\boldsymbol{r}'\right|\right) + \sum_{paths-j} i\left(\frac{2}{\pi}\left|\frac{\partial^2 L_j(\boldsymbol{r},\boldsymbol{r}')}{\partial r_\perp \partial r'_\perp}\right|\right)^{1/2} \exp\left[ikL_j(\boldsymbol{r},\boldsymbol{r}') + i\pi(n-1/4)\right]\right]$$

Modified RCM

J. Hart et al., PHYSICAL REVIEW E 80, 041109 (2009)

Original RCM: $$\underline{\underline{Z}}^{cav} = j\,\mathrm{Im}\left(\underline{\underline{Z}}^{rad}\right) + \left[\underline{\underline{R}}^{rad}\right]^{1/2} \cdot \underline{\underline{\xi}} \cdot \left[\underline{\underline{R}}^{rad}\right]^{1/2}$$

Modified RCM: $$\underline{\underline{Z}}^{cav} = j\,\mathrm{Im}\left(\underline{\underline{Z}}^{ave}\right) + \left[\underline{\underline{R}}^{ave}\right]^{1/2} \cdot \underline{\underline{\xi}} \cdot \left[\underline{\underline{R}}^{ave}\right]^{1/2}$$

Here Z^{ave} is the cavity impedance averaged over a prescribed frequency band
- **can be measured**
- **can be calculated if enough geometry is known**

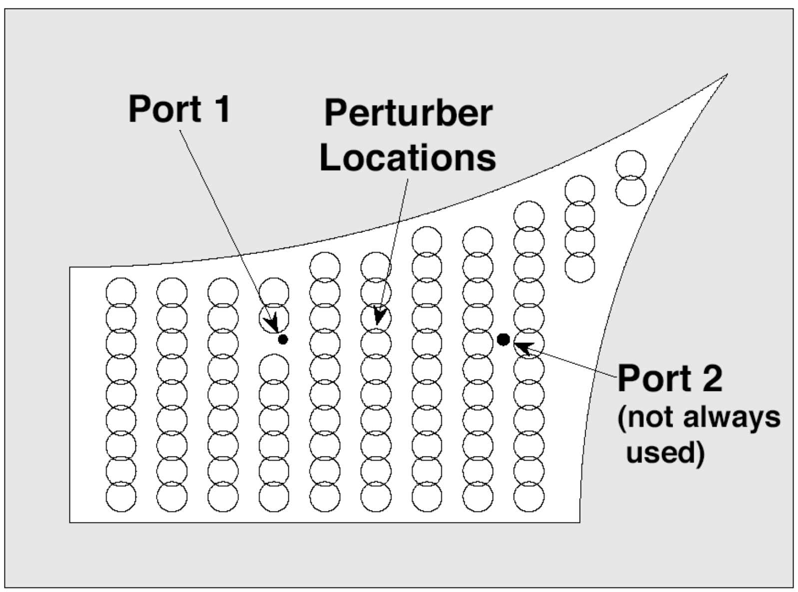
Calculated in geometric optics limit: $$\underline{\underline{Z}}^{ave} = j\,\mathrm{Im}\left(\underline{\underline{Z}}^{rad}\right) + \left[\underline{\underline{R}}^{rad}\right]^{1/2} \cdot \underline{\underline{\varsigma}} \cdot \left[\underline{\underline{R}}^{rad}\right]^{1/2}$$

$$\left(\underline{\underline{\varsigma}}\right)_{mn} = \sum_{\substack{ray\ paths-b \\ m\to n}} p_{b,mn}\, C_{b,mn} \exp[-jS_{b,mn}(\omega) - j\pi/4]$$

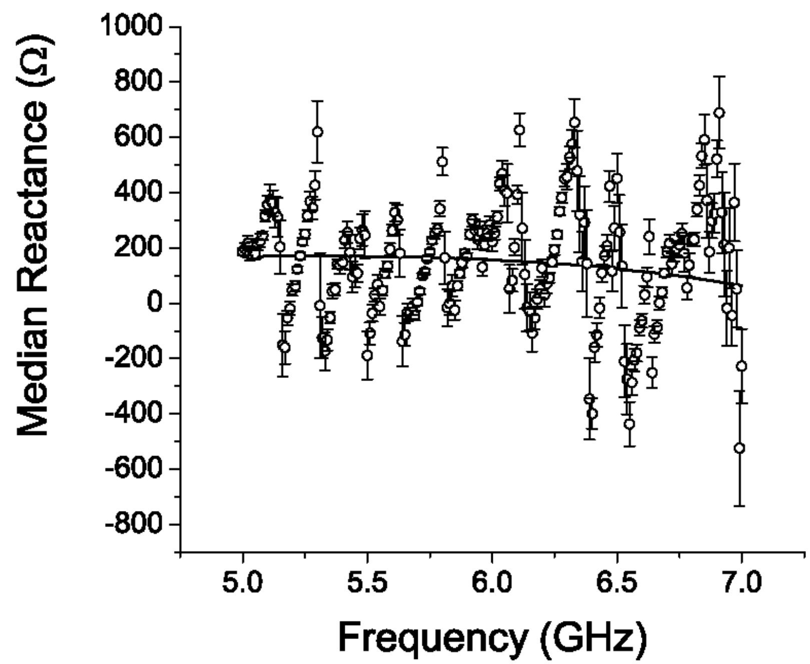
$p_{b,mn}$-attenuation, $C_{b,mn}$ - ray defocusing, $S_{b,mn}$ ray phase change=$kL_{b,mn}$

Numerical Tests (HFSS) on Bow Tie Cavity

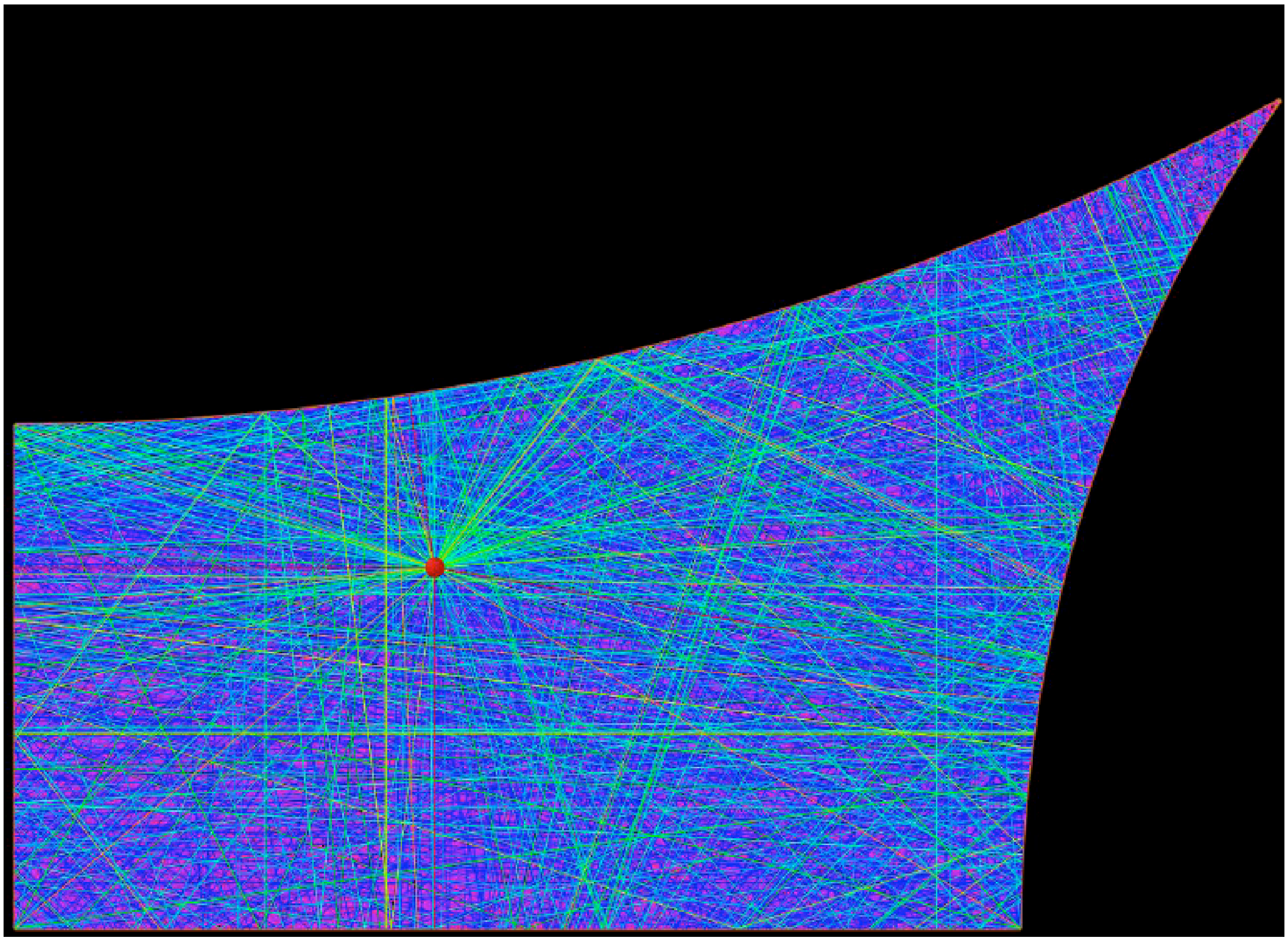
95 realizations generated by moving a circular perturber



Average (over realizations) impedance seen at port #1

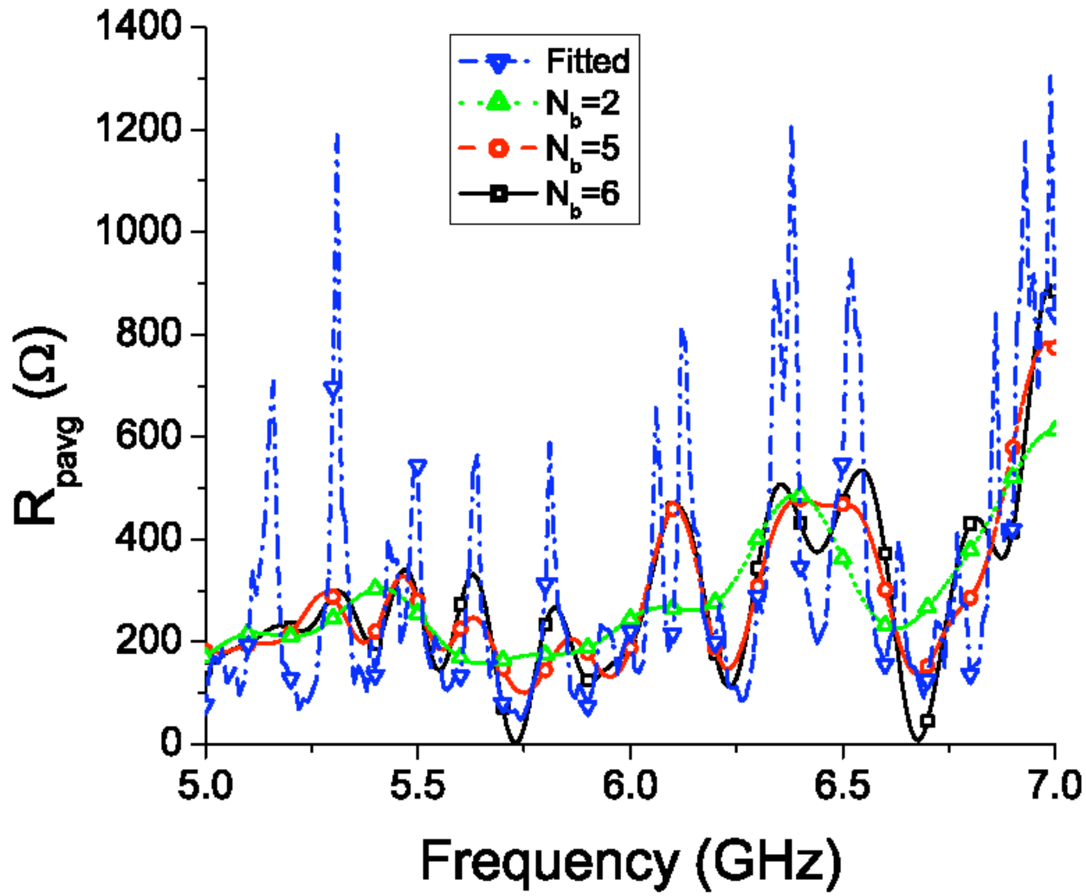


Multiple Bounces in the Bow Tie



Prediction of Average Resistance

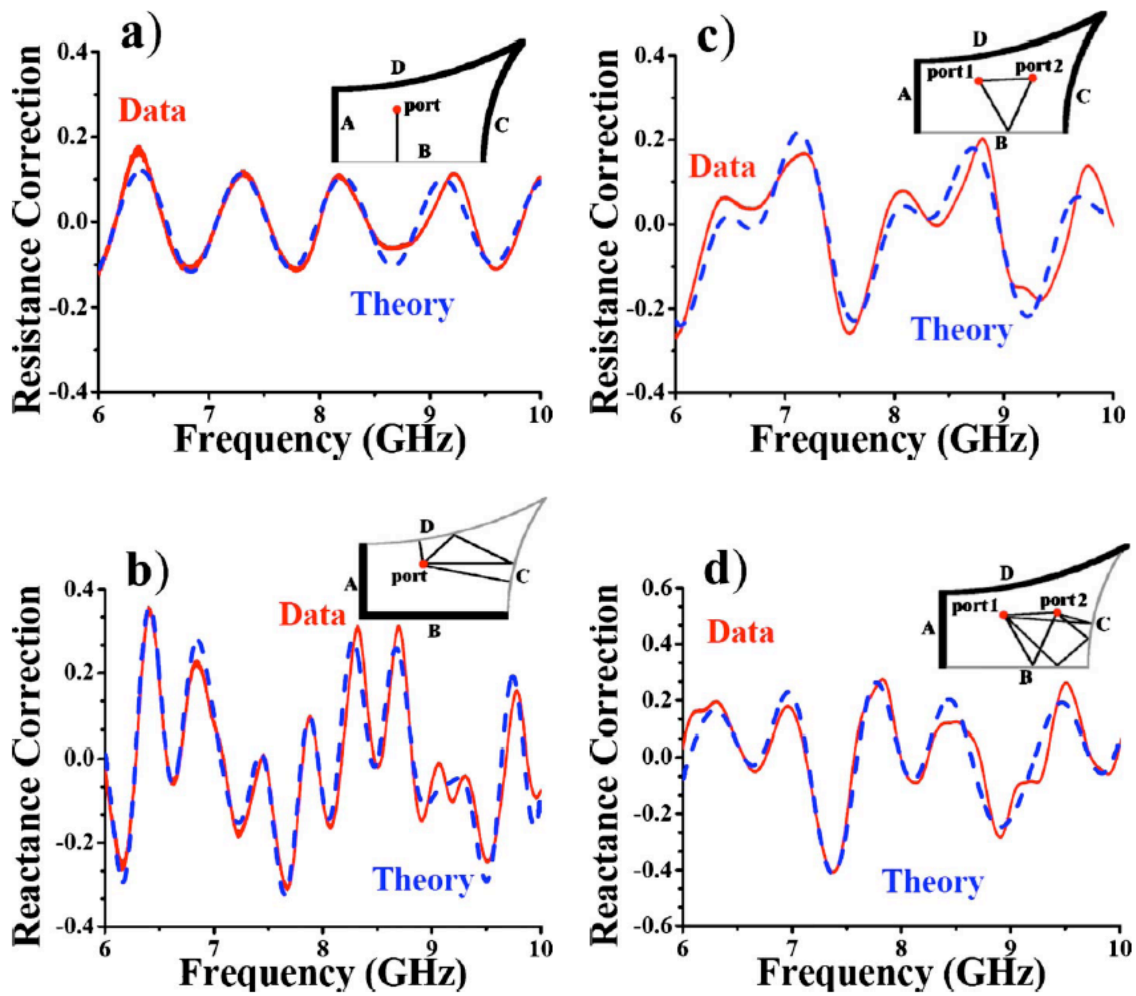
Theory includes contributions from N= 2, 4, 6 bounces.



Measurement of Effect of Short Orbits

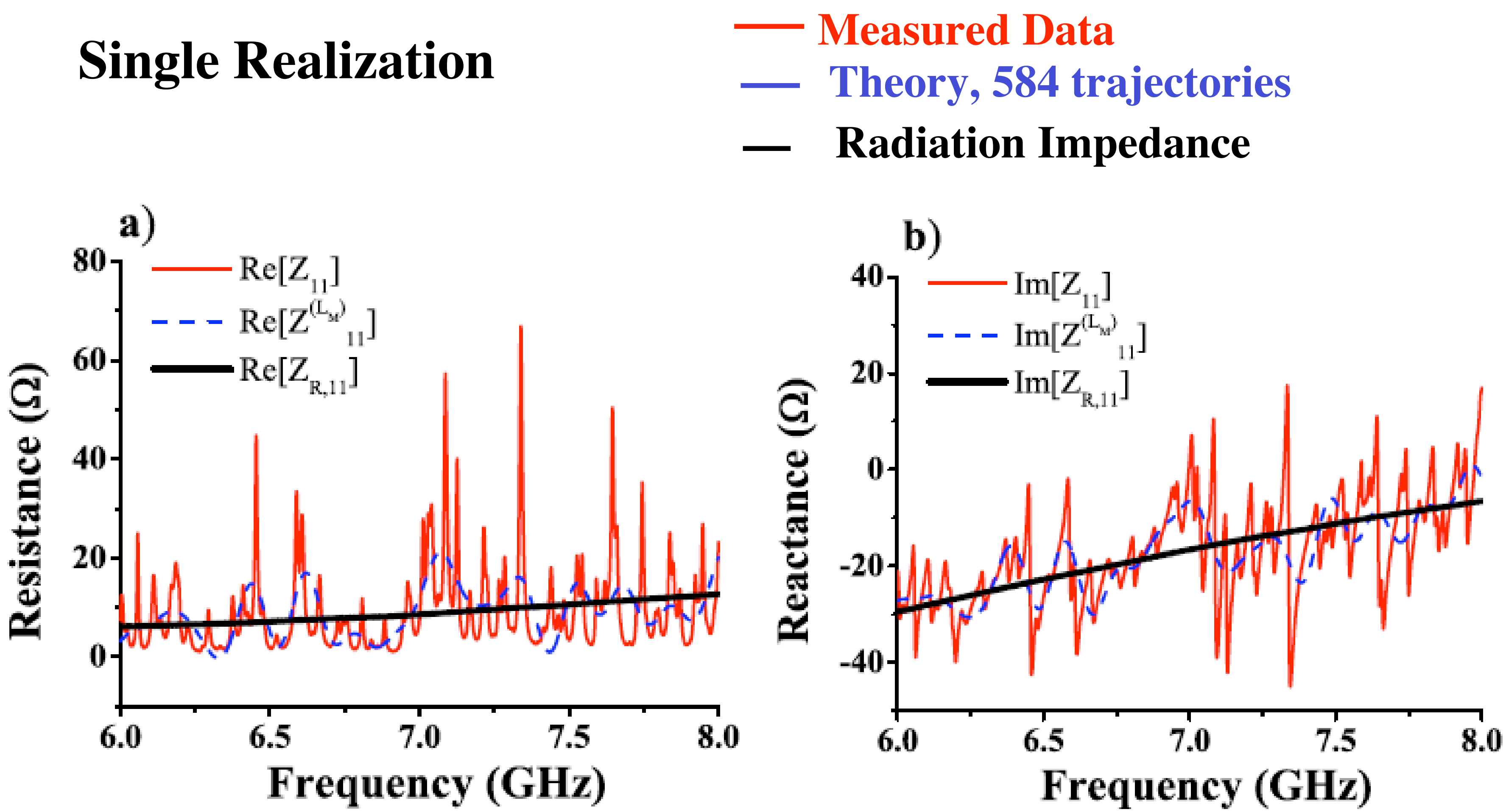
Yeh et al. PRE 82, 041114 (2010).

$$z_{correction} = \frac{Z - Z_{Rad}}{R_{Rad}}$$



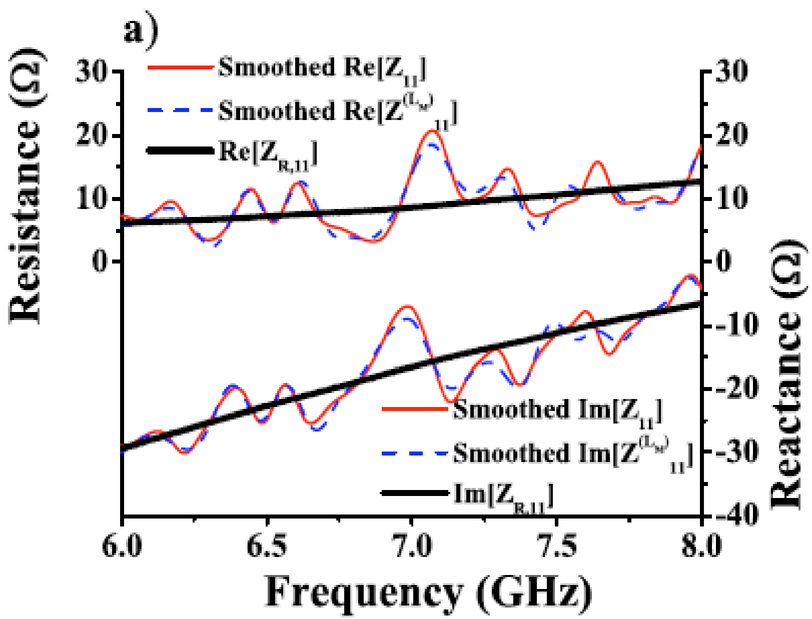
Measurement of Z11

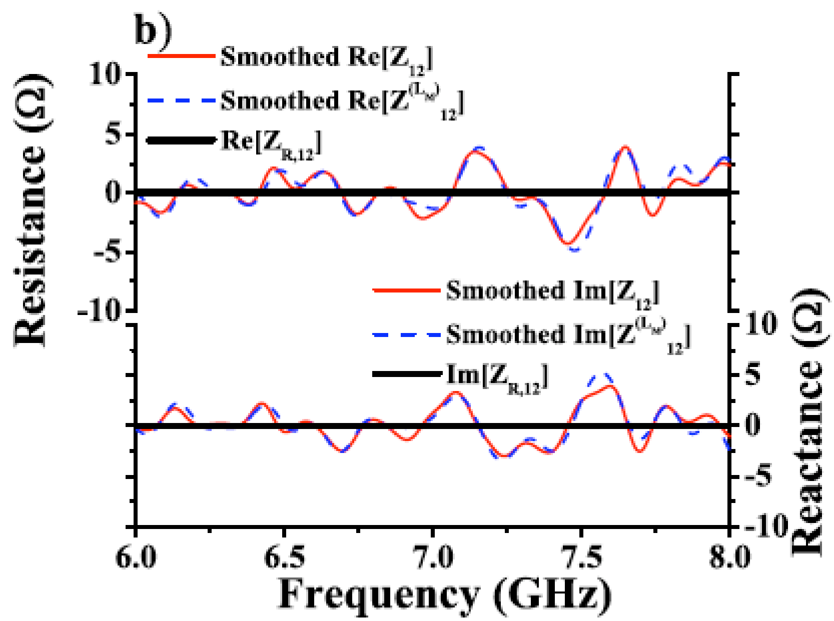
Single Realization



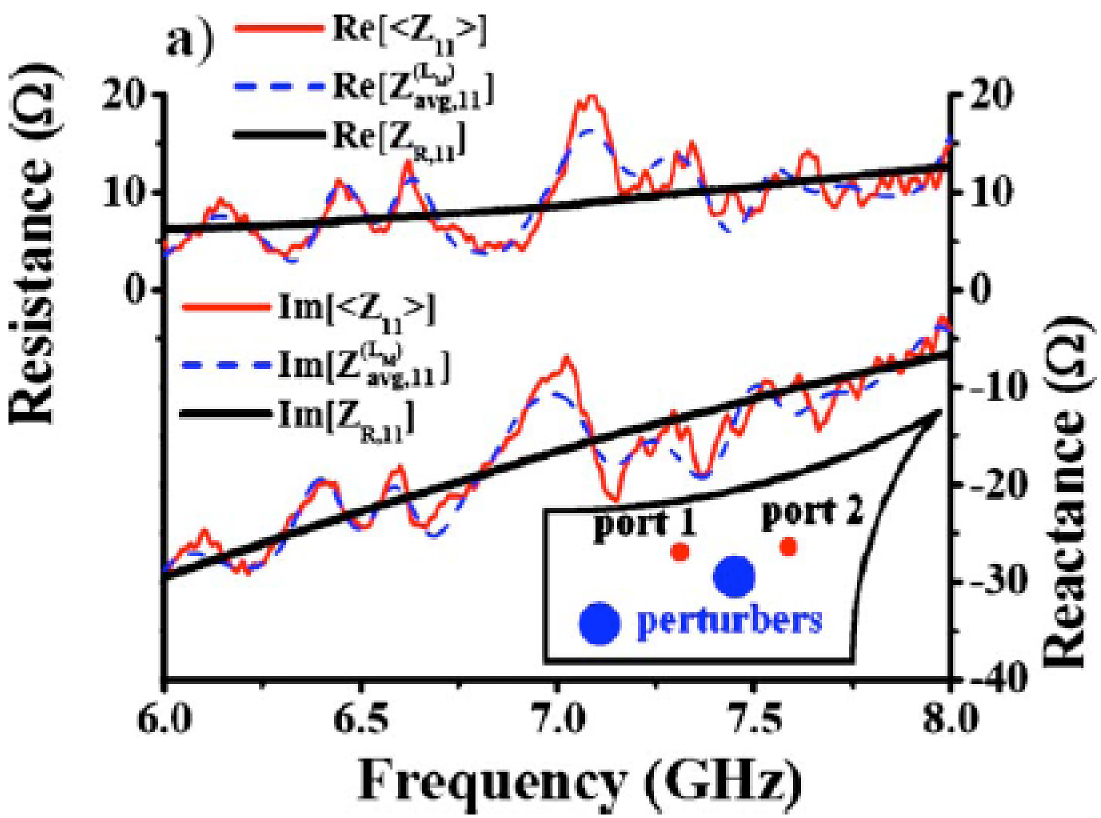
# Frequency Smooth Data
Window = 240 MHz

Single Realization





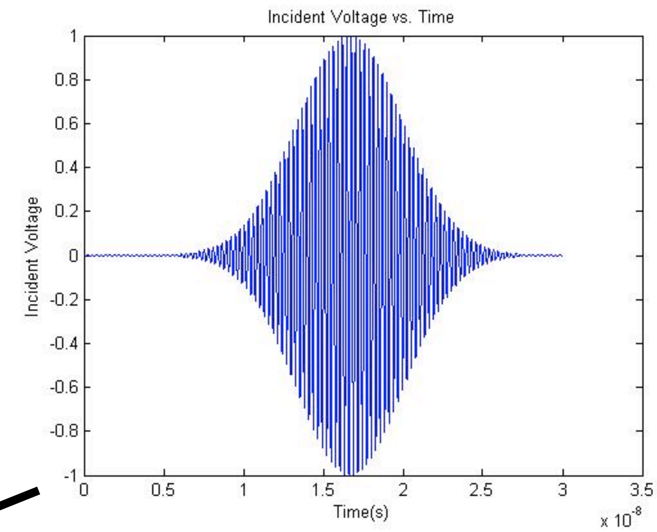
Configuration Averaging
100 Realizations



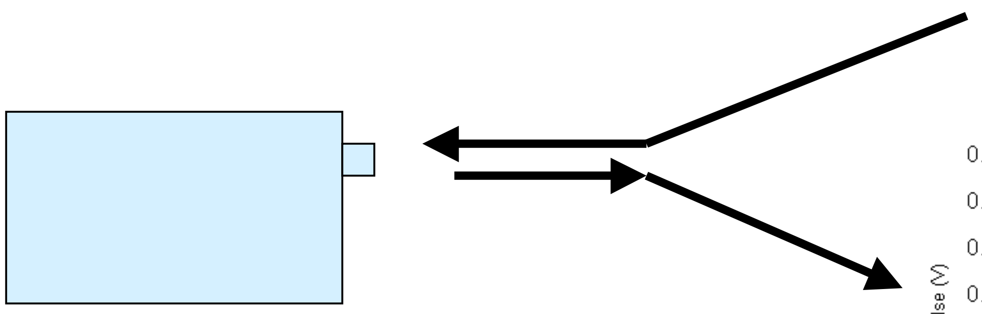
Contributions to theory from trajectories blocked by perturber are dropped.

Time Decay of Cavity Radiation

Question: What are the characteristics of the time decaying pulse emerging from the cavity?

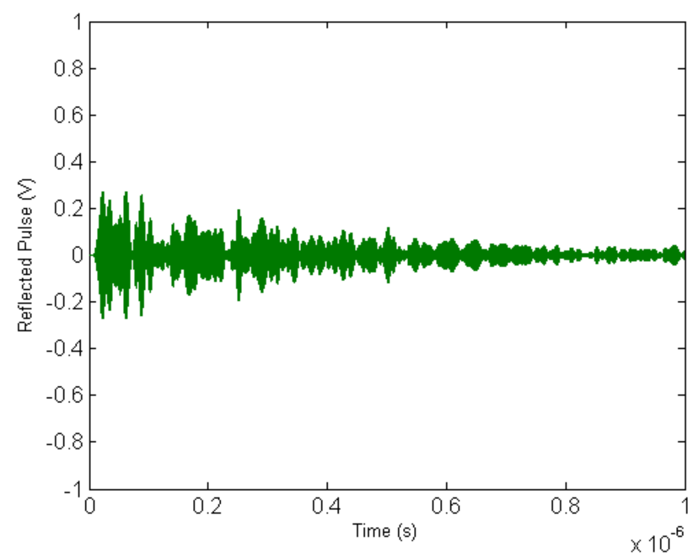


Incident Pulse



Complex cavity

Reflected Pulse
“Coda”



Long-Time Properties of Reflected Pulse

Hart et al. PRE 79, 016208 (2009)

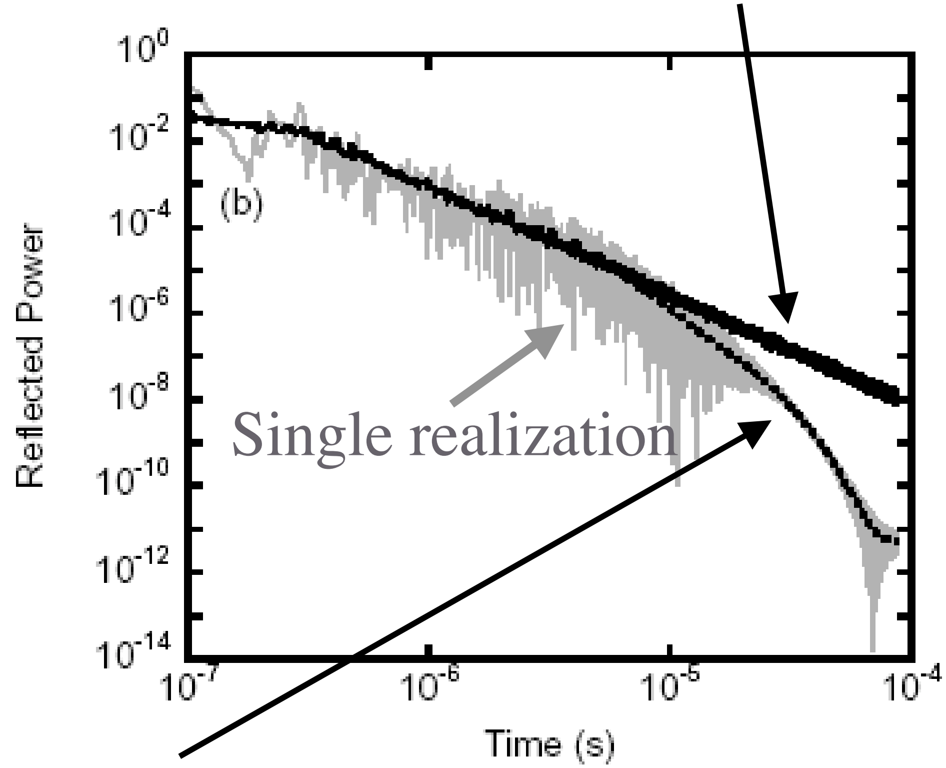
Output signal is a superposition of modes

$$V(t) = \sum_n V_n \exp\left[j(\omega_n + \gamma_n)t \right]$$

The following quantities have random distributions, determined by Random Matrix Theory (RMT)

$$V_n, \quad \omega_n, \quad \gamma_n$$

Ensemble average



Time average of single realization

General Results for Reflected Power

J. Hart et al. PRE to be published

1. Ensemble average decays with power law (already known)

2. Time average of an individual realization follows ensemble average for some time, and then decays exponentially

3. Individual realizations oscillate about their time average.

4. Transition from algebraic to exponential decay obeys universal distribution (parameters: pulse bandwidth, cavity mode spacing, port coupling strength)

Universal Transition for Gaussian Pulses

J. Hart et al. PRE (2009)

Ensemble Average

$$\bar{P}(t) \sim t^{-5/2}$$

$C(\alpha,\tau)$ = fraction of realizations with

$$P(t) < \alpha \bar{P}(t)$$

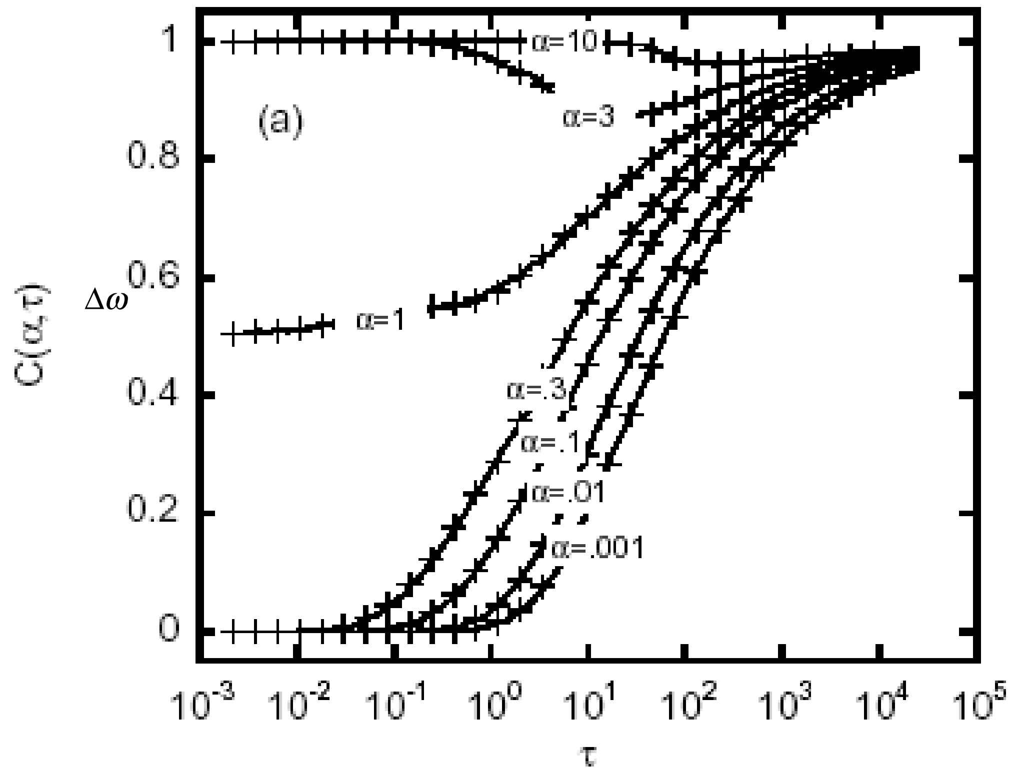
Normalized time

$$\tau = \frac{t\Delta\omega^3}{\omega_B^2 P_0}$$

$\Delta\omega$ -mode spacing

ω_B -pulse bandwidth

Theory for two different pulse widths



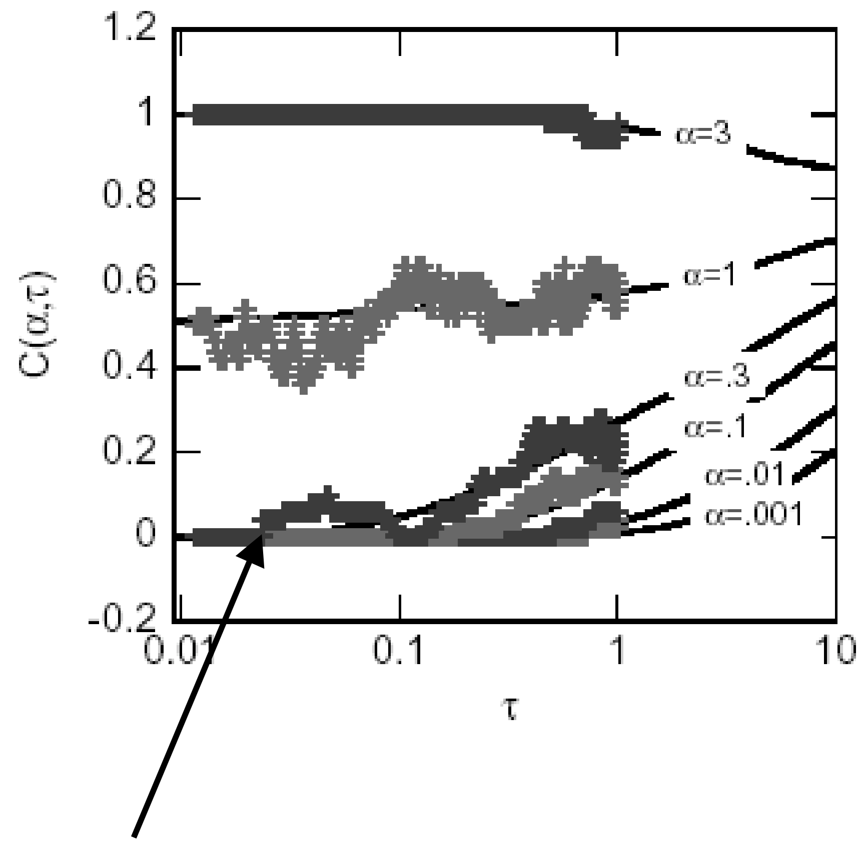
P_0 -depends on port coupling

Numerical verification via simulation

$C(\alpha,\tau)$ = fraction of realizations with

$$P(t) < \alpha \bar{P}(t)$$

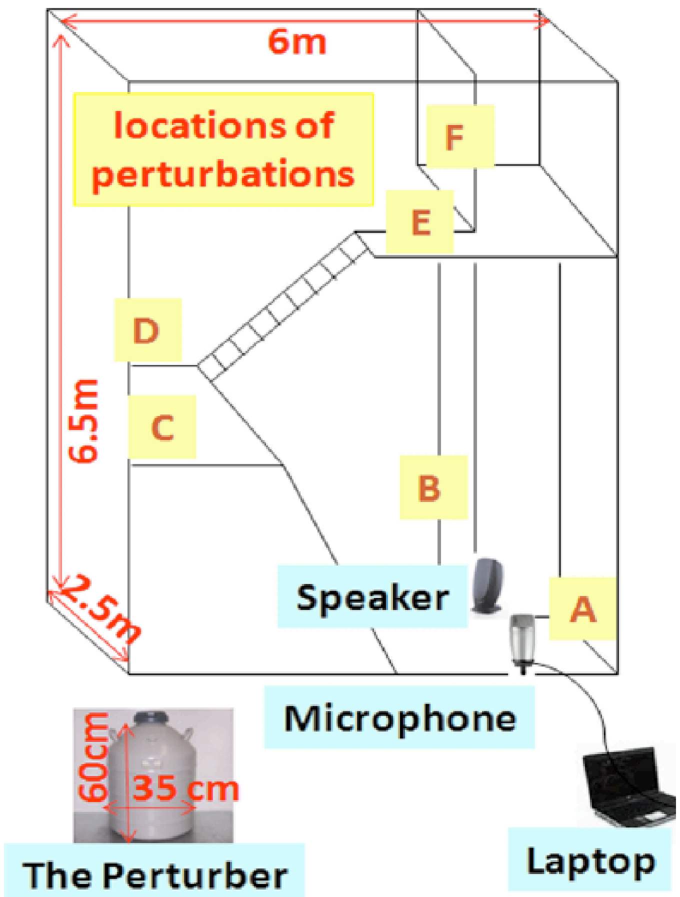
$$\tau = \frac{t\Delta\omega^3}{\omega_B^2 P_0}$$



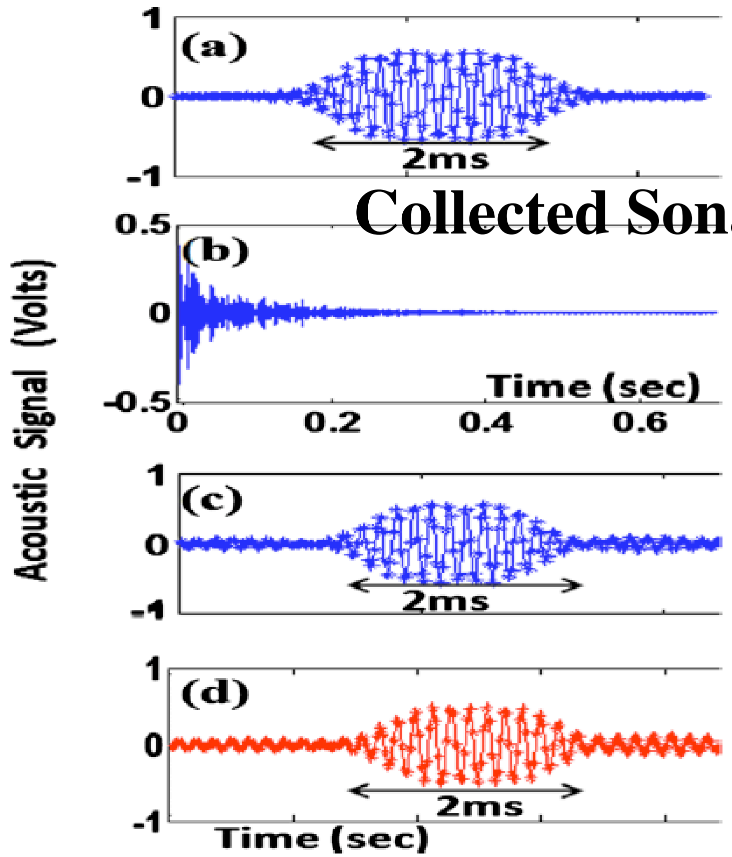
50 realizations of time domain equations

Sensing small changes in a wave chaotic scattering system

B. Taddese, J. Ap. Phys. 108, 114911 (2010)



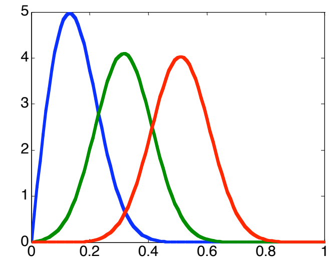
Initial Broadcast



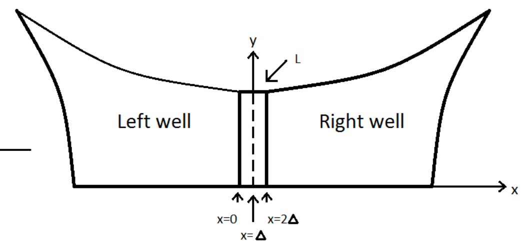
Reconstructed signals

Recent Developments

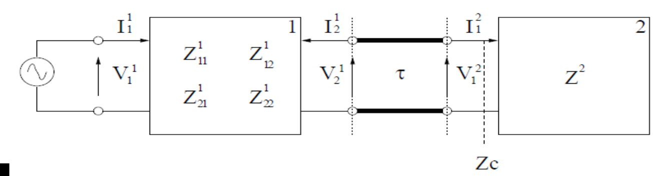
1. **First Principles derivation of fading distributions**



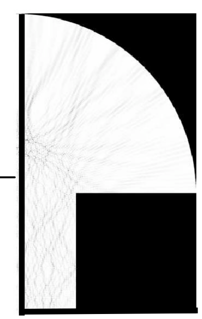
2. **Coupling of cavities**



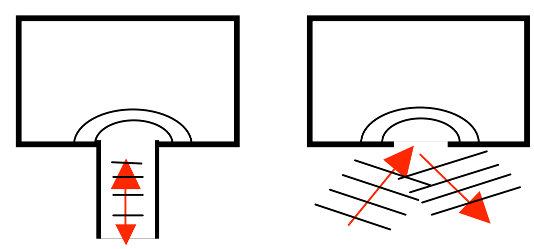
3. **Statistics of fields in chains of coupled cavities**



4. **Statistics of fields in mixed systems**



5. **Coupling by apertures**



Conclusions

Frequency Domain:

Universal (RMT) and system specific contributions to statistics of wave scattering.
Effects of losses, antennas, direst ray paths can be included.

Time Domain:

Decay of wave energy exhibits universal behavior
Time reversal symmetry can be used sensitivly determine changes in an environment